The Side Effects of Safe Asset Creation*

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Abstract

We present an incomplete markets model to understand the costs and benefits of increasing government debt when an increased demand for safety pushes the natural rate of interest below zero. A higher demand for safe assets causes the ZLB to bind, increasing unemployment. Higher government debt satiates the demand for safe assets, raising the natural rate, and restoring full employment. However, this entails permanently lower investment, which reduces welfare, since our economy is dynamically efficient even when the natural rate is negative. Despite this, increasing debt until the ZLB no longer binds raises welfare when alternative instruments are unavailable. Higher inflation targets instead allow for negative real interest rates and achieve full employment without reducing investment.

Keywords: safe assets, negative natural rate, crowding out, risk premium, liquidity traps
JEL Codes: E3, E4, E5, G1, H6

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1 Introduction

The most striking macroeconomic fact of the last three decades has been the dramatic decline in real interest rates in the United States and other advanced economies. This decline in rates saw many advanced economies pushed to the zero lower bound (ZLB), leaving conventional monetary policy unable to prevent a deep and lasting recession. This decline in safe rates was accompanied by a widening of spreads between the returns on risky and safe assets (Duarte and Rosa, 2015; Farhi and Gourio, 2018). In this context, many commentators have argued that an increase in the supply of safe assets – in particular, an increase in government debt – might be desirable from a welfare point of view because it would increase the natural rate of interest above zero, restoring the potency of conventional monetary policy (Caballero and Farhi, 2016). Indeed, a recent literature documents that changes in the supply of U.S. Treasuries affects the spread between safe and risky returns (Krishnamurthy and Vissing-Jorgensen, 2012)[1] - implying that an increase in the supply of U.S. treasuries could narrow spreads and raise the safe rate above zero. Our goal is to understand the cost and benefits associated with such a policy.

We present an analytically tractable model to evaluate the welfare effects of an increase in government debt in an environment which matches the stylized facts described above. The model features overlapping generations (OLG), idiosyncratic capital income risk, nominal rigidities and a ZLB. Importantly, risk generates a wedge between the safe rate of interest and the marginal product of capital, implying that the safe rate can be negative even when the economy is dynamically efficient. An increase in the supply of safe assets (government debt) can increase the safe rate because Ricardian equivalence does not hold.

The welfare implications of an increase in debt depend crucially on the presence of risk. We show this by first studying a model without nominal rigidities. We refer to allocations in this benchmark as "*natural allocations*", with the understanding that there is a continuum of natural allocations corresponding to different levels of government debt. An increase in the riskiness of the return on capital reduces real interest rates, as households attempt to substitute away from risky capital towards safe debt; a large enough increase in risk pushes interest rates below zero.[2] Higher government debt can offset this decline in the *natural rate of interest*[3] by satiating the demand for safe assets. But while higher debt insures old households against increased risk, it also crowds out investment in physical capital.

How this decline in capital affects social welfare depends on whether the economy is dynamically efficient. In an economy without risk, negative interest rates indicate that the economy is *dynamically inefficient*, i.e. there is an overaccumulation of capital (Diamond, 1965). In such an environment, increasing debt to restore a positive real interest rate is costless: it does crowd out capital, but crowding out *increases* social welfare. In our economy, in contrast, high enough risk can push the safe interest rate below zero while the net marginal product of capital remains positive and the economy remains *dynamically efficient*. In this scenario, increasing debt to restore positive interest rates is costly: it crowds out capital and crowding out

[1] Krishnamurthy and Vissing-Jorgensen (2012) argue that U.S. Treasuries enjoy a *convenience yield* reflecting their liquidity and safety attributes, which responds to the aggregate supply of Treasury debt. Del Negro et al. (2017) find that liquidity and safety premia are the main drivers of the secular decline in the natural rate.

[2] We study an increase in capital income risk because this is a tractable way of pushing the natural rate of interest below zero while generating an increase in spreads. However, the main tradeoffs we study would also be present if technological or demographic slowdown was responsible for a decline in the natural rate as in Eggertsson and Mehrotra (2014) as long as the economy is dynamically efficient.

[3] By the *natural rate of interest*, we mean the real interest rate arising in the natural allocation. Fiscal policy can determine the natural rate via the choice of government debt.

reduces social welfare. Absent nominal rigidities, this cost means it is never optimal to prevent real interest rates from falling below zero even though higher debt helps households self-insure. For intermediate levels of risk, the *optimal natural allocation* features no safe asset creation even though this entails negative real rates. For a high level of risk, the insurance benefits outweigh the costs of crowding-out, and the optimal natural allocation features positive debt - but still not enough to make interest rates positive.

However, in an economy with nominal rigidities and a binding ZLB, there is an additional benefit of increasing government debt when the natural rate is negative. As in the natural allocation, an increase in risk induces households to substitute away from risky capital towards safe government debt. But with monetary policy constrained by the ZLB, the interest rate cannot fall to clear the bond market. Higher demand for safe assets reduces demand for capital and consumption, causing prices to fall. With downward nominal wage rigidity, deflation raises real wages, persistently reducing employment. This in turn reduces the expected marginal product of capital and discourages investment, leading to a permanent slump.

An increase in government debt satiates the demand for safe assets without requiring negative rates, relaxing the ZLB and restoring full employment. This short-circuits the adverse feedback loop between unemployment and low investment, resulting in higher steady state capital than would occur without an increase in the supply of safe assets. Thus, even in our dynamically efficient economy, increasing debt until the ZLB no longer binds raises welfare. But the resulting level of capital is still lower than in the *optimal natural allocation*, which featured no safe asset creation and negative real rates. This leads to lower output and labor productivity even once the economy has returned to full-employment.

The fundamental problem is that the optimal natural allocation in a risky economy requires negative real rates to sustain high investment. When the ZLB binds, monetary policy cannot replicate this allocation. Safe asset creation presents monetary policy with the easier task of implementing a *different, suboptimal* natural allocation with positive real rates. A higher inflation target would instead permit negative real rates, implementing the optimal natural allocation with high investment and full employment.[4] This forces us to reassess whether low safe rates indicate a *shortage of safe assets*, as is sometimes argued.[5] We formalize a safe asset shortage as a situation in which issuing more safe assets increases welfare. Whether low rates indicate a shortage in this sense depends on whether negative real rates are implementable.

Related Literature A large literature studies the consequences of the secular decline in safe rates of interest and the supply of safe assets. Closest to our analysis is Caballero and Farhi (2016) who study an endowment economy in which safe asset shortages can drive an economy to the ZLB, generating a persistent recession. Like us, they find that increasing government debt can prevent the recession. This policy raises the natural rate of interest $r^\star$, allowing monetary policy to achieve full-employment by equating the real interest rate r (which is constrained by the ZLB) to the natural rate . But in our model, this implements a permanently lower level of investment than in the optimal natural allocation, reducing the full-employment level of GDP. This side effect of safe asset creation is absent in Caballero and Farhi (2016)'s endowment economy. Consequently, policies such as safe asset creation which raise $r^\star$ are just as good as policies such as a higher inflation target which facilitate a negative real interest rate r - all that matters is that $r = r^\star$.

[4] To be clear, in our economy with downward nominal wage rigidity, higher inflation is costless, and a higher inflation target is strictly preferred to higher debt.If inflation was costly, it would be optimal to increase both debt and target inflation somewhat. Our point is not that higher inflation is optimal, it is that there are costs associated with increasing debt.

[5] See for example Gourinchas and Jeanne (2012), Caballero et al. (2017b), Gourinchas and Rey (2016).

Our analysis instead suggests that not all policies which set $r = r^\star$ are created equal. Higher inflation, which equates $r = r^\star$ at a lower level, is better than safe asset creation, which equates them at a higher level, because low rates are essential to sustain the efficient level of investment in a risky world.

Farhi and Maggiori (2017) and Gourinchas and Rey (2016) explore another tradeoff arising in an international setting: increased safe asset provision is intrinsically good because it can avert a liquidity trap, but may be restricted because issuing safe assets exposes a sovereign to self-fulling "confidence crises" or real appreciations. In our closed economy setting with lump sum taxes, these concerns are not present. We study a different trade-off: while issuing safe assets prevents liquidity traps, it crowds out investment.

Our paper also relates to the recent literature studying how a contraction in private borrowing constraints, rather than a shortage of safe assets, can push economies with nominal rigidities into a liquidity trap (Eggertsson and Krugman, 2012; Guerrieri and Lorenzoni, 2017). Guerrieri and Lorenzoni (2017) noted that government debt can completely offset such a shock.[6] Since these models abstract from capital, there are no trade-offs associated with increasing the supply of government debt, since it offsets private borrowing constraints without any cost in terms of crowding out capital. In our environment, instead, while government debt can prevent the ZLB from binding, this comes at the cost of crowding out capital.

We are not the first to study the interaction of public debt and the ZLB in economies with capital. Eggertsson and Mehrotra (2014) and Auclert and Rognlie (2016) study how a tightening of borrowing contraints or an increase in labor income inequality can reduce aggregate demand in an incomplete markets economy with nominal rigidities. Like us, they find that public debt issuance can restore full employment when monetary policy is constrained.[7] In their models, capital is riskless; public debt can accommodate higher desired savings but does not act on the risk premium since capital and bonds earn the same return in equilibrium. Our setup features a risky return on capital, which endogenously depends on the supply of safe assets. Thus, relative to these papers, we uncover how policymakers can manage the supply of public safe assets to target not just the real interest rate, but also the *risk premium*. Our approach to modeling capital income risk draws most closely on Angeletos (2007).[8]

Our paper contributes to a large literature which studies the optimal supply of public debt (Woodford, 1990; Aiyagari and McGrattan, 1998). In these papers, the potential benefit of public debt is that it relaxes private constraints. More recently, Angeletos et al. (2016) study optimal debt policy in a flexible price economy when debt provides liquidity services. Their Ramsey planner trades off the liquidity benefits of higher debt against the cost of raising interest rates, requiring higher distortionary taxes. In our economy, the government has access to lump sum taxes, so this cost is absent. Instead, the cost of issuing more debt is that it reduces investment relative to the optimal natural allocation;[9] the benefit is that it avoids liquidity traps. This trade-off is absent in the papers just discussed which study flexible-price models.

Blanchard (2019) also emphasizes that the costs and benefits of higher debt depend not just on the safe rate but also on risky rates. While he studies these tradeoffs in a real model, we also study an economy with nominal rigidities and a ZLB, and highlight an additional benefit of increasing public debt when the ZLB

[6] Bilbiie et al. (2013) demonstrated a similar result in a Eggertsson and Krugman (2012)-type model.

[7] Bacchetta et al. (2016) also study the interaction between government debt and capital in a liquidity trap in a flexible price economy. Like them, we show that safe assets crowd out capital even in a liquidity trap. Unlike them, we study an economy with nominal rigidities, providing a reason to increase the natural rate which is absent in their flexible price economy.

[8] Brunnermeier and Sannikov (2016), Di Tella (Forthcoming), Azzimonti et al. (2014) and many others have also exploited a similar framework.

[9] In this regard, our result is reminiscent of Yared (2013) who shows that while increasing government debt can in principle substitute for limited private credit, it is not optimal to do so since this distorts investment decisions.

binds. While it would never be optimal to issue enough debt to make the safe rate positive in Blanchard (2019), this can be beneficial in our economy with nominal rigidities if other policy tools are unavailable.

Finally, we treat government debt as a "safe asset" in the sense that its return does not covary with a household's marginal utility, unlike the return on capital, the other asset in our economy. This differs from other definitions of safe assets which emphasize liquidity, default risk and so forth.[10] The rest of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes natural allocations in our economy. Section 4 describes the effects of risk and safe asset creation. Section 5 concludes.

2 Model

Households Time is discrete. At each date t, a cohort of ex-ante identical individuals with measure 1 is born and lives for two periods. Each individual $j \in [0,1]$ has identical preferences given by:

$$\mathbb{U}(c_t^Y, c_{t+1}^O) = (1-\beta)\ln c_t^Y + \beta \mathbb{E}_t \ln c_{t+1}^O$$

where $\beta \in (0, 1/2)$. When young, each household is endowed with one unit of labor which it is willing to supply inelastically and earns a nominal wage W_t. The household also receives a lump-sum transfer T_t from the government. Young households can invest in two assets: *risky* capital and *safe* government debt. The budget constraints of a household can be written as:

$$\begin{aligned} P_t c_t^Y + P_t k_{t+1} + \frac{1}{1+i_t} B_{t+1} &= W_t l_t + P_t T_t && (1) \\ P_{t+1} c_{t+1}^O(z) &= P_{t+1} R_{t+1}^k(z) k_{t+1} + B_{t+1} && (2) \end{aligned}$$

where i_t is the nominal interest rate on government debt and $R_{t+1}^k(z)$ is the real return on capital earned by old household i at date $t+1$, which depends on a random variable z described below. A young household must decide how much to invest in capital without knowing the realization of z in the next period. Importantly, we assume this risk is uninsurable: households cannot trade Arrow securities contingent on the realization of z. Appendix A shows that the households' optimal decisions are described by:

$$\begin{aligned} c_t^Y &= (1-\beta)(\omega_t l_t + T_t) && (3) \\ k_{t+1} &= \beta \eta_t (\omega_t l_t + T_t) && (4) \\ \frac{b_{t+1}}{R_t} &= \beta(1-\eta_t)(\omega_t l_t + T_t) && (5) \end{aligned}$$

where $b_t = \frac{B_t}{P_t}$ denotes real debt, $\omega_t = \frac{W_t}{P_t}$ denotes the real wage, $R_t = \frac{(1+i_t)P_t}{P_{t+1}}$ is the real return on government debt and η_t, the portfolio share of risky capital, is defined by

$$\eta_t \equiv \frac{k_{t+1}}{k_{t+1} + b_{t+1}/R_t} = \mathbb{E}_z \left[\frac{R_{t+1}^k(z) k_{t+1}}{R_{t+1}^k(z) k_{t+1} + b_{t+1}} \right] \quad (6)$$

[10] E.g., Gorton and Ordonez (2013) define safe assets as *information-insensitive* assets. Azzimonti and Yared (2017), He et al. (2019) and Farhi and Maggiori (2017) define a safe asset as one which has no default risk. In Barro et al. (2014), safe assets are the riskless bonds issued by less risk averse agents to more risk averse agents.

Young households consume a fraction $1-\beta$ of labor income net of transfers when young and save the rest. Out of the β fraction saved, households invest a fraction η_t in risky capital and $1-\eta_t$ in safe bonds. Appendix A shows that the optimal η_t solves a portfolio choice problem maximizing risk-adjusted returns:

$$\eta = \underset{\eta_t \in [0,1]}{\operatorname{argmax}} \mathbb{E}_z \ln \left[\eta_t R^k_{t+1}(z) + (1-\eta_t) R_t \right]$$

Agents demand more safe assets (lower η) if bonds are relatively cheap (low $1/R_t$) or risk is high:

Lemma 1 (Portfolio Choice)**.** *The optimal portfolio choice η_t depends negatively on R_t. Compare two distributions of $R^k(z)$, F and G where G is a mean-preserving spread of F. Then $\eta_F > \eta_G$.*

Proof. See Hadar and Seo (1990). □

Firms At each date t, each old household operates a firm with a Cobb-Douglas production technology $Y_t(z) = (z_t k_t)^\alpha (\ell_t(z))^{1-\alpha}$ where k_t is the amount of capital that household i invested when young. z is the firm-specific productivity and is i.i.d across all firms with distribution $\ln z \sim N\left(-\frac{\sigma_t^2}{2}, \sigma_t^2\right)$. Importantly, there is no market for capital among old households so households with low z cannot sell their capital to those with high z. Given its productivity and capital, the firm hires labor in order to maximize profits:

$$R^k_t(z) k_t = \max_{\ell} (z k_t)^\alpha \ell_t^{1-\alpha} - \omega_t \ell_t$$

where ω_t denotes the real wage. Labor demand is given by $\ell_t(z) = \left(\frac{1-\alpha}{\omega_t}\right)^{\frac{1}{\alpha}} z k_t$ and we can write the return to capital as $R^k_t(z) = \alpha \left(\frac{1-\alpha}{\omega_t}\right)^{\frac{1-\alpha}{\alpha}} z$.[11]

Government At date t, the government issues non-defaultable nominally safe one period debt B_{t+1} at price $1/(1+i_t)$, using the proceeds to repay outstanding debt B_t and disburse transfers $P_t T_t$ to the young:

$$\frac{1}{1+i_t} B_{t+1} = B_t + P_t T_t \tag{7}$$

The monetary authority sets nominal interest rates i_t according to some rule which we specify later.

2.1 Natural Allocations

Our ultimate goal is to consider how risk and the supply of safe assets interact with monetary policy in the presence of nominal rigidities. To this end, in Section 4 we will introduce nominal rigidities by assuming that nominal wages are sticky downwards but flexible upwards. However, in order to understand outcomes in the economy with nominal rigidities, it will be instructive to compare these to the outcomes arising in an economy with flexible prices and wages. To this end, we spend the remainder of Sections 2 and 3 characterizing allocations in such a benchmark economy, which we call *natural allocations.*

Labor Market In the benchmark economy, wages adjust to achieve full employment:

$$l_t = 1 \text{ and } \omega_t = (1-\alpha) k_t^\alpha \tag{8}$$

[11] We assume full depreciation of capital for simplicity.

Return on capital Given equilibrium wages (8), the return to investing in capital can be written as:

$$R_t^k(z) \quad = \quad \alpha z k_t^{\alpha-1} \tag{9}$$

Throughout, we will refer to increases in σ as *increases in risk*. Since $\ln z \sim N(-\sigma^2/2, \sigma^2)$, an increase in σ is a mean-preserving spread to the distribution of idiosyncratic productivity, leaving the average return on capital (9) unchanged.

Goods Market Clearing The aggregate resource constraint of this economy can be written as:

$$c_t^Y + \int_z c_t^O(z) dF_t(z) + k_{t+1} = \int_z (z k_t)^\alpha \, \ell_t(z)^{1-\alpha} dF_t(z) = k_t^\alpha \tag{10}$$

where $F_t(z)$ is the cdf of the log-normal distribution defined above. The LHS of the equation above is the sum of total consumption and investment in capital in period t while the RHS is GDP.

Definition 1 (Equilibrium without nominal rigidities)**.** *Given a sequence $\{B_{t+1}, i_t, T_t\}_{t=0}^\infty$ and initial conditions $\{B_0, k_0\}$, an equilibrium is a sequence $\{c_t^Y, c_t^O(z), k_{t+1}, l_t, \ell_t(z), R_t^k(z), P_t, W_t\}_{t=0}^\infty$ such that*

1. *$\{c_t^Y, c_t^O(z), k_{t+1}, B_{t+1}\}$ solves households' problem, given prices $\{i_t, R_t^k(z), P_t, W_t\}$ and transfers $\{T_t\}$*
2. *$\{\ell_t(z), R_t^k(z)\}$ solve the firm's problem at each date t*
3. *government budget constraint (7), labor market clearing (8), goods market clearing (10) are satisfied.*

In this economy without nominal rigidities, the classical dichotomy holds and we can discuss relative prices and allocations without reference to nominal variables. We refer to equilibrium allocations in such an economy as *natural allocations*, and call the prevailing real interest rate R_t the *natural rate of interest*. Importantly, there are many natural allocations in our economy, and they depend on fiscal policy, in particular on the path of government debt. There are two key equations that help us describe the dynamics of the economy in any natural allocation:

Aggregate supply of capital The first of these equations is the *aggregate supply of capital*, which can be derived by combining the expression for labor income (8), government budget constraint (7) and transfers from (4) and (5):

$$k_{t+1} = \beta \left[(1-\alpha) k_t^\alpha + \frac{b_{t+1}}{R_t} - b_t \right] - \frac{b_{t+1}}{R_t} \tag{11}$$

where the LHS of (11) denotes the total savings in the economy at date t, minus bond purchases.

Demand for capital The other equation of interest concerns the demand for capital which is described by the optimal portfolio choice of young households:

$$\eta_t \quad = \quad \mathbb{E}_z \left[\frac{\alpha z k_{t+1}^\alpha}{\alpha z k_{t+1}^\alpha + b_{t+1}} \right] = \mathbb{E}_z \left[\frac{\alpha z}{\alpha z + \tilde{b}_{t+1}} \right] \tag{12}$$

where $\tilde{b}_t = \frac{b_t}{k_t^\alpha}$ denotes the debt-to-GDP ratio. This expression can be derived by plugging in (9) into (6). (12) shows that the equilibrium portfolio share of capital only depends on capital and bonds only via debt-to-GDP. In what follows, it will be convenient to work with $\tilde{b}$ instead of b as our measure of fiscal policy. Also, Jensen's inequality implies that η_t is decreasing in σ. Finally, using the expression for η_t, the *demand for capital* can be expressed as (see Appendix B for details):

$$\alpha k_{t+1}^{\alpha-1} \quad = \quad g(\tilde{b}_{t+1}, \sigma) R_t \qquad \text{where} \qquad g(\tilde{b}, \sigma) = \frac{\mathbb{E}_z \left[(\alpha z + \tilde{b}_{t+1})^{-1} \right]}{\mathbb{E}_z \left[z(\alpha z + \tilde{b}_{t+1})^{-1} \right]} > 1 \tag{13}$$

Capital demand is decreasing in the safe real interest rate. However, it also depends on the supply of safe assets and the level of idiosyncratic risk. Since the LHS of (13) is the expected return on capital $\mathbb{E}_z R_{t+1}^k(z)$, $g(\tilde{b}_{t+1}, \sigma)$ can be interpreted as the risk premium earned by capital relative to bonds. $g(\tilde{b}_{t+1}, \sigma)$ is increasing in σ. An increase in risk, σ, decreases the demand for capital and widens the spread between the expected return on capital and the safe rate. As capital becomes riskier, investors would like to substitute away from capital towards government debt; if no increase in the supply of debt is forthcoming, either the price of debt must rise or investment in capital must fall. $g(\tilde{b}_{t+1}, \sigma)$ is also decreasing in $\tilde{b}_{t+1}$: increasing $\tilde{b}_{t+1}$ reduces the safety premium by satiating the demand for safe assets. In this sense, our model provides a micro-founded channel through which the supply of public safe assets affects the risk premium as found empirically by Krishnamurthy and Vissing-Jorgensen (2012), although here the premium reflects safety rather than liquidity.

The intersection of the aggregate supply of capital (11) and the demand for capital (13) determines the equilibrium level of investment k_{t+1} and real interest rates R_t given today's capital stock and government debt policy. Capital accumulation in any natural allocation, given a sequence $\{\tilde{b}_{t+1}\}_{t=0}^\infty$, is described by:

$$k_{t+1} = s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma) k_t^\alpha \qquad \text{where} \qquad s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma) = \frac{\beta(1 - \alpha - \tilde{b}_t)}{\beta + (1-\beta)\mathbb{E}_t \left[\dfrac{\alpha z}{\alpha z + \tilde{b}_{t+1}} \right]^{-1}} \tag{14}$$

This equation is reminiscent of the Solow model, with the aggregate savings rate given by $s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma)$.[12] The savings rate is decreasing in both the current period and next period's debt-to-GDP ratio. Higher $\tilde{b}_t$ requires higher taxes on young savers, reducing their disposable income and thus the amount they save. High $\tilde{b}_{t+1}$ tomorrow, in equilibrium, requires that young households hold more bonds in their portfolio, reducing the amount they invest in capital. Finally, higher risk induces young savers to shift their portfolio away from riskier capital towards safe government debt, reducing aggregate savings.

Steady State In steady state, the aggregate supply of capital (11) becomes:[13]

$$k^{1-\alpha} = \beta(1-\alpha) - \left[\frac{1-\beta}{R} + \beta \right] \tilde{b} \tag{15}$$

[12] Note that the aggregate savings rate is different from the private savings rate of the young which is given by β.
[13] Here and elsewhere, quantities and prices without time sub-scripts denote steady state values.

Equation (15) shows that government debt crowds out capital, diverting savings away from investment in capital, and (if $R > 1$) increasing taxes on young savers. Issuing zero debt maximizes steady state capital. Given debt to GDP, (15) defines an increasing relation between capital and the interest rate, depicted by the upward sloping red curves in both panels of Figure 1: higher interest rates make the same amount of debt cheaper for young savers, leaving ample funds available for investment. The downward sloping blue curves depict the demand for capital (13). The intersection of the two curves determines capital and interest rates in the steady state of the natural allocation with steady state debt-to-GDP $\tilde{b}$:

$$k\left(\tilde{b},\sigma\right) = \left[\frac{\beta(1-\alpha-\tilde{b})}{\beta+(1-\beta)\mathbb{E}\left[\frac{\alpha z}{\alpha z+\tilde{b}}\right]^{-1}}\right]^{\frac{1}{1-\alpha}} \qquad \text{and} \qquad R\left(\tilde{b},\sigma\right) = \frac{1}{1-\alpha-\tilde{b}}\left[\beta^{-1}\mathbb{E}\left[\frac{1}{\alpha z+\tilde{b}}\right]^{-1}-\tilde{b}\right] \quad (16)$$

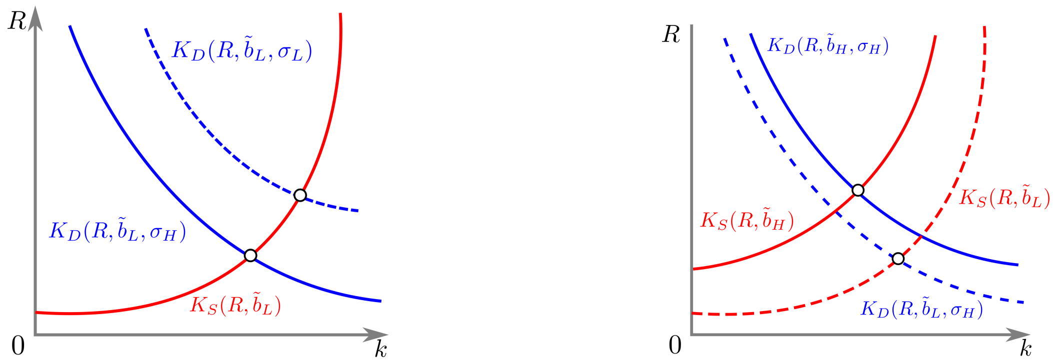
3 Inspecting the Mechanism

We now show that an increase in risk can reduce the natural rate of interest, while an increase in the supply of safe assets can increase the natural rate (which is the same as the *prevailing* real interest rate, without nominal rigidities). Prior to date 0, capital and the natural rate are at their steady state levels, $k(\tilde{b}_L,\sigma_L)$, $R(\tilde{b}_L,\sigma_L)$. At date 0, σ increases unexpectedly and permanently from σ_L to $\sigma_H > \sigma_L$.

Can the natural rate be negative in the absence of risk? Even before we consider *changes* in σ or $\tilde{b}$, equation (16) reveals that even in the riskless case with $\sigma = 0$, the steady state natural rate can be negative for small enough $\tilde{b}$. For instance, with $\tilde{b} = 0$, $R(0,0) = \frac{\alpha}{\beta(1-\alpha)}$, which could be less than 1 even in the absence of risk. In this case, the economy would be *dynamically inefficient* in the sense of Diamond (1965). For the rest of the paper, we rule this out.

Assumption 1. *The riskless economy is dynamically efficient:* $\frac{\alpha}{\beta(1-\alpha)} > 1$

An increase in risk (σ) is a convenient way to generate negative safe rates in our model. In a richer model, a fall in productivity growth or population growth could also push the natural rate below zero. The main trade-offs associated with safe asset creation when the natural rate is negative would be the same if the negative natural rate was caused by demographic or technological slowdown, rather than an increase in risk – provided that the economy features a positive *level* of risk, and the riskless economy is dynamically efficient (as in Assumption 1). This is because, as shown in Appendix C, Assumption 1 ensures that the economy remains dynamically efficient when an increase in risk, or fall in productivity or population growth drives the safe rate below zero, so crowding out reduces welfare. We study the effects of an increase in risk since it affords us a tractable way of generating a fall in safe rates (even into negative territory) along with widening spreads, as documented by Duarte and Rosa (2015); Farhi and Gourio (2018) and others.[14]



(a) A permanent increase in σ

(b) A permanent increase in $\tilde{b}$

Figure 1. Steady States

3.1 The effects of an increase in risk

We now show that an increase in risk can reduce the natural rate of interest. To build intuition, assume bonds are in zero net supply, $\tilde{b} = 0$, but $\sigma > 0$. In this case, the steady state natural rate (16) becomes $R = \frac{\alpha}{\beta(1-\alpha)} e^{-\sigma^2}$. Ceteris paribus, an increase in the riskiness of capital causes young households to demand more safe assets; with no increase in the supply of safe assets forthcoming, their price must rise. In particular, if risk σ^2 exceeds $\underline{\sigma}^2 \equiv \ln\left[\frac{\alpha}{\beta(1-\alpha)}\right] > 0$, the steady state natural rate is negative. This decreasing relationship between risk and natural rates holds more generally for $\tilde{b} > 0$.

Lemma 2. *For a given level of $\tilde{b}$, the steady state level of capital k is weakly decreasing in σ while the steady state natural rate of interest R is strictly decreasing in σ.*

To see this, apply Jensen's inequality to (16). Higher risk makes households substitute away from risky capital towards safe bonds. Given a fixed supply of bonds, their price $1/R$ must rise to clear the market. Facing higher prices of safe assets, young savers, who save a fixed fraction β of their total income, have less left over to invest in capital. Thus the aggregate saving rate and capital stock fall. Figure 1a depicts this graphically. An increase in σ shifts the capital demand schedule leftwards (moving from the downward sloping dashed blue line to the solid blue line) while leaving the aggregate supply of capital (upward sloping red curve) unchanged, reducing steady state capital and real interest rates. Importantly, a high enough σ can result in a negative natural rate in steady state, $R < 1$.

Figure 2 depicts the dynamics of the economy following the permanent increase in risk from σ_L to σ_H at date 0. The solid red line depicts when fiscal policy keeps $\tilde{b}$ constant at the same level $\tilde{b}_L$ as before date 0. The increase in risk pushes real interest rates immediately into negative territory and also reduces the capital stock, as the economy converges to the new high risk steady state.

3.2 The effects of an increase in safe assets

While risk can depress the natural rate, an increase in the supply of safe assets always increases the natural rate. However, this crowds out investment, reducing steady state capital.

[14] Duarte and Rosa (2015) present evidence from a variety of asset pricing models that the equity risk premium increased significantly between 2000-13. Del Negro et al. (2017) find that liquidity and safety premia are the main factors explaining the secular decline in the natural rate. Caballero et al. (2017a) and Farhi and Gourio (2018) also find that the real return on capital remained flat or increased over the past three decades, while the return on U.S. Treasuries declined dramatically.

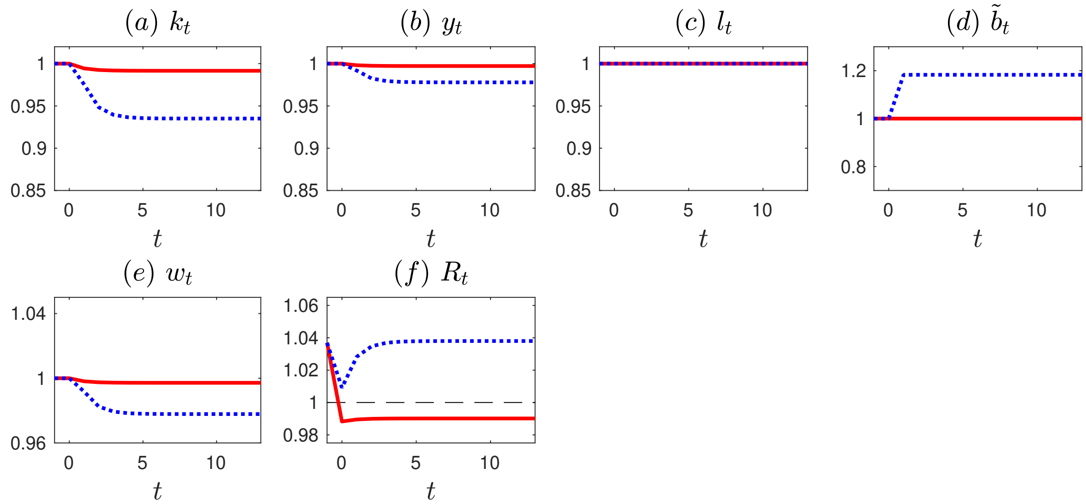


Figure 2. Dynamics. Solid red lines denote the natural allocation following the permanent increase in σ while the dotted blue lines denote the natural allocation with an increase in safe assets following the increase in risk.

Lemma 3. *The steady state level of capital k is strictly decreasing in $\tilde{b}$ while the steady state real interest rates R is strictly increasing in $\tilde{b}$.*

Figure 1b depicts this graphically. An increase in government debt satiates the demand for safe assets and reduces the safety premium in (13). Consequently, young households are willing to hold more capital for a given real rate, shifting the capital demand schedule right (from the downward sloping dashed blue curve to the solid blue curve). However, higher government debt diverts savings away from capital crowding out investment, shifting the aggregate supply of capital to the left (from the upward sloping dashed red curve to the solid red curve). Overall, steady state capital is unambiguously lower, and real interest rates higher, with a higher supply of safe assets. Thus in response to any increase in risk, a large enough increase in the supply of safe assets can always keep the natural rate positive – at the cost of crowding out investment.

The dashed blue lines in Figure 2 depict the dynamic when fiscal policy permanently increases the supply of safe assets from $\widetilde{b}_L$ to a higher level $\widetilde{b}_H$ starting from date 0. The Figure shows that while this increase in the supply of safe assets keeps the real interest rate positive throughout the transition to the new steady state, it results in a larger decline in capital stock, output and wages.

3.3 The optimal natural allocation

By increasing debt, fiscal policy can always implement a natural allocation with a positive natural rate. Just because policy *can* do this does not mean that it *should*. As we now show, a planner who maximizes steady state welfare would *not* create enough safe assets to prevent negative interest rates. We consider a social planner who maximizes steady state welfare:

$$\mathbb{W}\left(k, \widetilde{b}\right) \equiv (1-\beta) \ln \left[(1-\alpha-\tilde{b})k^{\alpha} - k\right] + \beta \mathbb{E}_z \ln \left[(\alpha z + \tilde{b})k^{\alpha}\right] \tag{17}$$

subject to (16) by choosing $k \geq 0, \tilde{b} \geq 0$. The planner faces the following trade-off. In equilibrium, an increase in government debt is essentially a forced transfer from the young to the old.[15] Since the planner

[15] Here debt is financed by lump sum taxes on the young. If in addition there were lump sum taxes on the old, our results would be unchanged, if we redefine b_t as government debt net of taxes on the old. In this sense, $\tilde{b}$ can be broadly interpreted

cannot directly insure old people against low realizations of z, she can only raise their consumption in low z states via an unconditional transfer. This provides an *insurance motive* for creating safe assets. But if the old consume a greater share of GDP, the young must consume a smaller share. The more risk old households face, the higher the expected marginal utility of the average old individual, and thus the stronger the insurance motive, i.e. the gains from redistribution from young to old. However, safe asset production also crowds out physical capital investment. As long as the economy is dynamically efficient, this harms both the young, who earn lower wages, and the old, who earn less capital income. Absent crowding out, the planner would create just enough debt that the real interest rate is zero.

Lemma 4. *Consider the unconstrained problem in which the planner maximizes (17), ignoring the constraint. The solution to this problem is unique, with either $R \geq 1$ and $\widetilde{b} = 0$, or $R = 1$ and $\widetilde{b} > 0$.*

Proof. See Appendix D. □

Intuitively, R measures how much an individual values consumption when young relative to when old. If risk is low, impatience outweighs the desire to insure against consumption risk when old, and a unit of consumption is worth more when young than when old, i.e. $R > 1$. But when risk is high, individuals would willingly forgo one unit of consumption when young to receive one unit when old, i.e. $R < 1$. The planner shares the individuals' preferences, but (unlike them) has a technology which transfers one unit of consumption from young to old, namely government debt. Thus the planner would never permit $R < 1$; that would signal an unmet desire for transfers from young to old, which could be satiated with more debt.

However, safe asset creation does crowd out investment. Thus, it is in fact *not constrained optimal* to produce enough safe assets to keep real interest rates positive, as we now show.

Proposition 1 (Constrained optimal natural allocation)**.** *If the riskless steady state is dynamically efficient, there exist $\overline{\sigma} \in (\underline{\sigma}, \infty)$ such that the solution to (17) has the following properties:*

i. If risk is low enough, i.e. $\sigma \leq \underline{\sigma}$, then the planner chooses $\widetilde{b} = 0$, $R \geq 1$.

ii. If risk is in the intermediate range $\sigma \in (\underline{\sigma}, \overline{\sigma}]$, the planner still does not choose to create safe assets and her optimal choices satisfy $\widetilde{b} = 0$, $R < 1$.

iii. If risk is high, $\sigma > \overline{\sigma}$, then the planner chooses to create some safe assets, but not enough to make real interest rates positive. The optimal choices satisfy $\widetilde{b} > 0$, $R < 1$.

In cases (ii) and (iii) the constrained planner creates strictly less safe assets than in the unconstrained problem.

Proof. See Appendix E. □

Intuitively, consider the net social benefit from a marginal increase in b starting from $b = 0$ when $\sigma = \underline{\sigma}$, so $R = 1$ absent any safe asset creation. This net benefit is

$$\underbrace{\beta \mathbb{E}_z \frac{1}{c^O(z)} - (1-\beta)\frac{1}{c^Y}}_{\substack{\text{net benefit of transferring \$1} \\ \text{from young to old}}} + \underbrace{\frac{\partial \mathbb{W}}{\partial k}}_{\substack{\text{social benefit} \\ \text{of more capital}}} \underbrace{\left(\frac{dk^{ss}(b)}{db}\right)}_{\text{crowding out}} \tag{18}$$

as private holdings of safe assets plus public transfers to the old.

This net benefit is negative when $b = 0$ and $\sigma = \underline{\sigma}$. In the private economy $\beta R \mathbb{E}_z \frac{1}{c^O(z)} - (1-\beta)\frac{1}{c^Y} = 0$. When $\sigma = \underline{\sigma}$, $R = 1$, and the first term in (18) is zero. That is, creating safe assets does not directly (i.e., holding capital fixed) increase welfare, to first order, when $\sigma = \underline{\sigma}$. Issuing debt transfers resources from young to old, but the average marginal utility of young and old is already equalized, since $R = 1$.

While creating safe assets has no direct effect on welfare, to first order, it does indirectly reduce welfare by crowding out capital. There are two steps in this argument: safe assets crowd out capital $\left(\frac{dk^{ss}(b)}{db} < 0\right)$, and this is costly because capital is socially valuable ($\frac{\partial \mathbb{W}}{\partial k} > 0$). While we have discussed the first point at length, the second point deserves more discussion. The first order effect of capital on social welfare is

$$\frac{\partial \mathbb{W}}{\partial k} = (1-\beta)\frac{1}{c^Y}\frac{d\omega}{dk} + \beta \mathbb{E}_z\left[\frac{1}{c^O(z)}\frac{dR^k(z)}{dk}k\right] > (1-\beta)u'(c^Y)\left[\frac{d\omega}{dk} + \mathbb{E}_z \frac{dR^k(z)}{dk}k\right] = 0$$

By the Envelope Theorem, a reduction in the capital stock only affects welfare via its effect on factor prices. Lower investment in capital by the young involves a *pecuniary externality*, reducing wages but increasing the rate of return to capital. Since all income is earned by either labor or capital, the *average* increase in capital income for the old is exactly offset by the fall in labor income for the young. Since $R = 1$, the expected marginal utility of old and young agents is equal, so this redistribution from young to old would not change welfare if the gain in capital income was enjoyed by all old agents equally. However, $\frac{d^2 R^k(z)}{dkdz} < 0$: capital income increases more for those with a high z and therefore a lower marginal utility.[16] Thus, the gain in expected utility of the old is outweighed by the loss in utility of the young. In other words, the pecuniary externality associated with a lower capital stock increases the share of risky income and reduces the share of safe income, lowering welfare in this incomplete markets economy.[17,18]

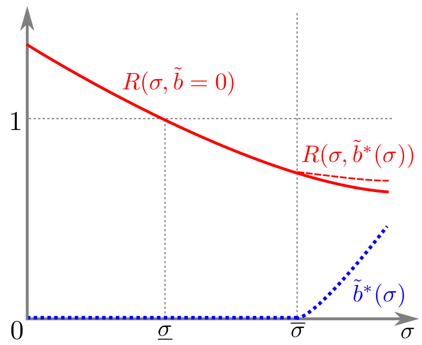
Figure 3a illustrates $\tilde{b}$ in the optimal natural allocation as a function of σ (dotted blue line) and the corresponding natural rate (red line). When risk is low ($\sigma \leq \underline{\sigma}$), natural rates are positive even with zero debt, and there is no insurance benefit from safe asset creation; even an unconstrained planner would not issue debt. For intermediate risk ($\sigma \in (\underline{\sigma}, \overline{\sigma}]$), natural rates are negative with zero debt, but welfare is still maximized by keeping debt at zero (solid red curve in Figure 3b): the insurance benefits are outweighed by the costs of crowding-out. When risk is high enough ($\sigma > \overline{\sigma}$), the optimal natural allocation features some debt (welfare is maximized at some $\widetilde{b} > 0$, as shown by the dashed blue curve in Figure 3b) – but not so much that the natural rate becomes positive. Thus, the natural rate in the optimal natural allocation (dashed red line), lies above the natural rate with zero debt (solid red line), but below $R = 1$ (horizontal line). As R approaches 1, the insurance benefit vanishes, and the costs outweigh the benefits.

The reason that increasing government debt is generally undesirable even when real rates are negative is that it crowds out capital, and crowding out reduces social welfare. This might seem counterintuitive, since negative interest rates typically – in the absence of risk – indicate dynamic inefficiency and capital over-accumulation (Diamond, 1965). In a dynamically inefficient economy, higher debt would crowd out

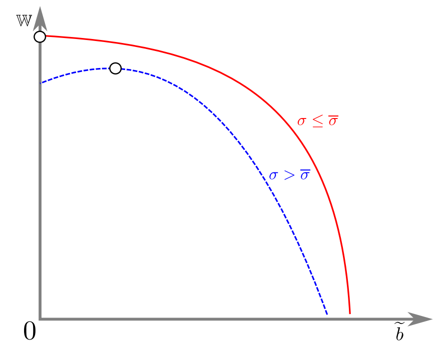
[16]To see this, note that $R^k(z) = z\alpha k^{\alpha-1}$ and so $\frac{d^2R^k(z)}{dkdz} = \alpha(\alpha-1)k^{\alpha-2} < 0$.

[17]This result is similar to Davila et al. (2012) who find that an appropriately calibrated Aiyagari (1994) economy features under-accumulation of capital from the perspective of a utilitarian planner: higher capital would raise wages and depress returns on capital, benefiting poor individuals who hold less capital.

[18]Appendix E.1 shows that the main result in Proposition 1 extends to an economy with homothetic time-separable utility and less than full depreciation. More generally, the appendix shows that with arbitrary concave preferences and neoclassical production technology, debt crowds out capital in the neighborhood of any stable steady state, and this reduces welfare as long as $\frac{d^2R^k(z)}{dkdz} < 0$ and the riskless steady state features positive interest rates (i.e. the economy is dynamically efficient).



(a) Optimal $\tilde{b}$ as a function of σ



(b) Welfare as a function of $\tilde{b}$

Figure 3. Steady State Welfare

capital, but this would be a benefit, not a cost. However, in the presence of risk, the safe rate can be negative even when the economy is dynamically efficient (Abel et al., 1989).[19] (In our setting, this is guaranteed by Assumption 1.) To see how risk affects the net benefits of higher debt, rewrite (18) as:

$$\left[-\left(\frac{R-1}{R}\right)+(1-\alpha)\left(\mathbb{E}_z R^k(z)-1\right)\frac{dk^{ss}(b)}{db}\right]\left(\frac{1-\beta}{c^Y}\right) \tag{19}$$

While the first term in the square brackets (representing the direct benefit of transfers from young to old) is positive whenever the safe rate R is less than 1, the second term in the square brackets is negative in a dynamically efficient economy with $\mathbb{E}_z R^k(z) > 1$, indicating a loss from crowding out. In the absence of risk, we would have $R = \mathbb{E}_z R^k$ and the two terms would have the same sign. Blanchard (2019) presents a similar expression for the net benefit of higher debt in an OLG model with aggregate risk, and emphasizes that risky and safe rates are important in determining whether higher debt is beneficial. Our framework with idiosyncratic risk provides analytical tractability and allows us to describe the precise conditions under which higher debt is (or is not) optimal.

Our results are not driven by the fact that we have characterized optimal policy by maximizing steady state welfare. Appendix F characterizes the solution to the problem of a Ramsey planner who maximizes

[19] Defining dynamic efficiency is not straightforward in our environment with idiosyncratic risk. Phelps (1965) and Diamond (1965) study economies without risk, and label an equilibrium *dynamically inefficient* if there exists another sequence for the aggregate capital stock that produces more aggregate consumption in some periods and never produces less aggregate consumption. Under this criterion, our economy is never dynamically inefficient given Assumption 1. However, this definition is not very relevant in our heterogeneous agent economy with risk. Abel et al. (1989) study economies with aggregate risk, and present two definitions of dynamic efficiency which are equivalent in their setting (but not in ours). According to their first definition, an allocation is dynamically efficient if no generation's ex-ante welfare can be increased without reducing the ex-ante welfare of another. This definition makes sense in their setting with only aggregate risk, but not in our economy with uninsured idiosyncratic risk, because such Pareto improvements exist in any equilibrium – starting from any equilibrium allocation, an unconstrained planner could equalize consumption across all old agents in a given cohort, increasing each cohort's welfare.

According to Abel et al. (1989)'s second definition, an allocation is dynamically efficient if capital income is larger than investment, or equivalently if the expected net return on capital is positive. Our economy does satisfy this criterion: even when the safe rate is negative, the expected return on capital is positive because capital earns a risk premium.

Our preferred definition of *under-accumulation* of capital is instead the following. Consider a planner who can choose a path for the aggregate capital stock, but cannot redistribute between old agents with different realizations of productivity. Appendix G shows that, starting from a steady state with zero safe assets (which features the highest level of capital attainable in equilibrium), this planner can deviate to a path which increases ex ante welfare for each cohort and has higher capital in every period. While this deviation cannot be supported as an equilibrium in our setting, it highlights that this economy is not characterized by an inefficiently high level of capital. The reason, as discussed above, is that a higher capital stock raises wages (the safe component of income) and reduces capital income (the risky component of income), increasing welfare on net.

intertemporal welfare taking into account costs and benefits associated with transitions. Specifically, we consider the problem of a planner who puts arbitrary Pareto weights on the welfare of different cohorts, and define a *constrained efficient* allocation as one which is not Pareto dominated by any other allocation. Appendix F shows that constrained efficient allocations are similar to the characterization in Proposition 1. The natural allocation with zero debt is constrained efficient as long as risk remains below a certain level $\sigma^{\diamond} > \underline{\sigma}$ - even if the natural rate is negative ($\sigma \in (\underline{\sigma}, \sigma^{\diamond}]$). However, if risk is large enough, $\sigma > \sigma^{\diamond}$, the insurance benefits outweigh the cost of crowding-out, and the zero-debt policy is Pareto dominated.

4 Nominal rigidities

The analysis above revealed that optimal natural allocations feature zero safe assets even if risk pushes the natural rate below zero. However, in the presence of nominal rigidities and the ZLB, monetary policy may not be able to replicate this *optimal* natural allocation precisely because it involves negative real interest rates. This failure results in an downturn with persistently elevated unemployment. Safe asset creation increases the natural rate, presenting monetary policy with a natural allocation which it can attain without violating the ZLB. While this allows monetary policy to restore full employment, this comes at a cost since monetary policy is not attaining the optimal natural allocation.

To discuss this tradeoff, we introduce nominal rigidities by assuming nominal wages are sticky downwards but flexible upwards. Following Eggertsson and Mehrotra (2014) and Schmitt-Grohé and Uribe (2016), workers are unwilling to work for wages below a wage norm $\tilde{W}_t$; the prevailing wage is given by:

$$W_t = \max\left\{\widetilde{W}_t, P_t\omega_t^*\right\} \text{ where } \ln \widetilde{W}_t = (1-\gamma)\ln\left(\Pi^{\star} W_{t-1}\right) + \gamma \ln\left(P_t \omega_t^*\right) \tag{20}$$

$\Pi^{\star} \geq 1$ denotes the monetary authority's inflation target (described below) and $\omega_t^* = (1-\alpha)k_t^{\alpha}$ is the real wage that delivers full employment given capital k_t. $\gamma \in [0,1)$ measures wage flexibility. With $\gamma = 0$, nominal wages are rigid downwards; with $\gamma = 1$, wages are fully flexible. When nominal wages exceed the market clearing nominal wage $P_t\omega_t^*$, labor demand is less than supply, resulting in unemployment: $\int_0^1 \ell_{i,t} di < 1$. When there is unemployment, households are proportionally rationed, so each young household supplies the same amount of labor l_t. Firms are always on their labor demand curve and the prevailing nominal wage satisfies $W_t = (1-\alpha)k_t^{\alpha} l_t^{-\alpha} P_t$. This yields a relationship between employment and inflation:

$$l_t = \min\left\{ l_{t-1}^{1-\gamma}\left(\frac{k_t}{k_{t-1}}\right)^{1-\gamma}\left(\frac{\Pi_t}{\Pi^{\star}}\right)^{\frac{1-\gamma}{\alpha}}, 1\right\} \tag{21}$$

The labor market can be in one of two regimes. When last period's nominal wage lies below the wage that would clear markets today, and full employment requires nominal wages today to rise, wages jump to their market clearing level and and there is full employment, $l_t = 1$. However, when last period's wage lies above today's market clearing wage, and full employment requires wages to fall, the wage norm binds, and wages only partially fall towards their market clearing level, resulting in unemployment. In this unemployment regime, employment will be higher, all else equal, if it was higher last period (which signals that wages were not too high and don't have far to fall); if capital is higher today than last period (which means the market clearing wage is higher today than last period); or if current inflation is higher. Temporarily higher inflation greases the wheels of the labor market by reducing $\widetilde{W}/P$, lowering labor costs and increasing labor demand. Note that there is no money illusion in the long-run, since we include $\Pi^\star$ in (20): higher target inflation does not relax downward nominal wage rigidity. In sections 4.1 and 4.2 we normalize $\Pi^\star$ to 1 and in Section 4.3 we consider the effects of an increase in $\Pi^\star$.[20]

Monetary Policy Monetary policy sets nominal interest rates according to the following flexible inflation targeting rule subject to the ZLB:

$$\left(\frac{\Pi_t}{\Pi^\star}\right)\left(\frac{Y_t}{Y_t^\star}\right)^{\psi} \leq 1, \qquad i_t \geq 0, \qquad \left\{\left(\frac{\Pi_t}{\Pi^\star}\right)\left(\frac{Y_t}{Y_t^\star}\right)^{\psi} - 1\right\} i_t = 0 \tag{22}$$

where $Y_t^\star = k_t^\alpha$ is the level of output consistent with full employment, and $\Pi^\star$ is the monetary authority's inflation target. Intuitively, the monetary authority aims to implement the target of $\Pi^\star$ and full employment whenever the ZLB does not prevent this. The monetary authority is willing to tolerate above target inflation if employment is below target; ψ denotes the weight placed on output stabilization, relative to price stability.[21] When the ZLB constrains policy, however, both inflation and output may be below target.

Equilibrium with nominal rigidities The remaining equations characterizing equilibrium are similar to those characterizing natural allocations, except that the economy might not be at full employment. The aggregate supply of capital is given by:

$$k_{t+1} = \beta\left[(1-\alpha)k_t^\alpha l_t^{1-\alpha} + \frac{b_{t+1}}{R_t} - b_t\right] - \frac{b_{t+1}}{R_t} \tag{23}$$

With full employment ($l_t = 1$), (23) is identical to (11) (the supply of capital in the natural allocation). Unemployment today ($l_t < 1$) reduces the income of the young, reducing their savings and therefore demand for both capital and bonds. Similarly, savers' optimal portfolio decision (12) becomes $\eta_t = \mathbb{E}_z\left[\frac{\alpha z k_{t+1}^\alpha l_{t+1}^{1-\alpha}}{\alpha z k_{t+1}^\alpha l_{t+1}^{1-\alpha} + b_{t+1}}\right]$. As before, the equilibrium portfolio share of capital depends on the expected ratio of capital income to total income of the old. Unemployment reduces the marginal product of capital (MPK) and increases the portfolio share of safe assets (reduces η_t), given k_{t+1} and b_{t+1}. The demand for capital becomes:

$$\alpha\left(\frac{k_{t+1}}{l_{t+1}}\right)^{\alpha-1} = g(\tilde{b}_{t+1} l_{t+1}^{\alpha-1}, \sigma) R_t \tag{24}$$

where the average MPK is now $\mathbb{E}_z R_{t+1}^k(z) = \alpha\left(\frac{k_{t+1}}{l_{t+1}}\right)^{\alpha-1}$. The demand for capital in the natural allocation (13) is simply equation (24) with $l_{t+1} = 1$. Unemployment tomorrow ($l_{t+1} < 1$) lowers the average MPK, reducing capital demand for a given R_t. However, lower l_{t+1} also increases the portfolio share of safe assets, narrowing the spread between the safe rate on bonds and the risky return on capital. Intuitively, the consumption of the old contains a risky (capital) and a safe component (bonds). The higher the risky share of income, the higher the covariance of consumption and the return to capital and the higher the

[20] To ensure that the steady state is unique, we assume $\Pi^\star < \alpha^{-1}$. This is not a demanding restriction. The standard value for the capital share, α, is 1/3. Thus, our assumption states that the inflation target is less than 200 percent ($\Pi^\star < 3$).

[21] This can be thought of as the limit of a rule $1 + i_t = \max\left\{1, R^\star\Pi^\star\left(\frac{\Pi_t}{\Pi^\star}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_t^\star}\right)^{\phi_y}\right\}$ as $\phi_\pi \to \infty$ and $\phi_y/\phi_\pi \to \psi$.

risk premium demanded by the young. Higher future unemployment lowers the risky share, leaving old households less exposed to risk, and reducing the risk premium. Overall, the dynamics of the economy with nominal rigidities are described by equations (21)-(24).

4.1 The possibility of risk-induced stagnation

An increase in risk in the presence of nominal rigidities can result in persistent or even permanent unemployment, as we now show. As in Section 3, there is a permanent unanticipated increase in σ at date 0 from σ_L to $\sigma_H > \sigma_L$, where the corresponding steady state natural rates of interest satisfy $R(\tilde{b}_L, \sigma_L) > 1 > R(\tilde{b}_L, \sigma_H)$. For now, fiscal policy keeps $\tilde{b}_t$ constant at the same level $\tilde{b}_L$ as before date 0.[22] The following proposition describes equilibrium behavior of the economy from date 0 onwards.

Proposition 2 (Stagnation)**.** *Suppose $\tilde{b}_t = \tilde{b}_L$ for all $t \geq 0$ and for $t < 0$ the economy is in steady state with $R(\tilde{b}_L, \sigma_L) > 1$. At date 0, σ_t unexpectedly and permanently increases to σ_H with $R(\tilde{b}_L, \sigma_H) < 1$. Then:*

1. *There is no bounded equilibrium in which the economy returns to a steady state with full-employment.*
2. *For ψ sufficiently high and γ sufficiently low, there exists a unique equilibrium in which $i_t = 0$ for all $t \geq 0$ and the economy converges to a steady state with unemployment.*

Proof. See Appendix H. □

At date 0, young savers want to reallocate their portfolios away from increasingly risky capital towards safe government debt. With $\tilde{b}$ fixed, the excess demand for bonds necessitates a fall in the real return on bonds to equilibrate the market. Absent inflation, this requires a large cut in nominal interest rates, but the ZLB prevents this. Thus the real rate is *too high*, lowering the demand for capital, and thus the price of output (i.e. consumption and capital). With sticky nominal wages, the fall in price is only partially met by a fall in nominal wages, causing higher real wages, lower labor demand, and unemployment. The fall in young households' income reduces their demand for both bonds and capital – clearing the bond market, but reducing investment.

Next period, the capital stock is lower, reducing the marginal product of labor and hence labor demand. Since nominal wages adjust slowly to their market clearing level, unemployment persists and is expected to persist in the future. Anticipating a lower MPK, young households have even less reason to invest in capital – which is now permanently more risky – rather than safe bonds. With $\tilde{b}$ fixed, an excess demand for bonds persists, the ZLB prevents interest rates from falling to clear markets, and investment slumps further. Unemployment remains permanently high, since there is a permanent excess demand for safe assets (even with $i_t = 0$ forever), and so it takes permanently lower income to equate demand and supply.

Figure 4 depicts this graphically.[23] In the figure, k_{-1}, y_{-1}, w_{-1} and $\tilde{b}_{-1}$ are each normalized to unity. For comparison, the dashed gray lines illustrate the natural allocation.[24] With $\tilde{b}$ constant, higher risk pushes the natural rate of interest (shown in panel (f)) permanently below zero . While this causes a

[22] Recall that $\tilde{b}_t = b_t/k_t^\alpha$ is the ratio of debt to the the level of GDP in the natural allocation. $\tilde{b}$ might be smaller than the ratio of debt to actual GDP because of unemployment.

[23] In all figures, we set $\alpha = 1/3$, $\beta = 0.495$, $\sigma_L = 0.49$, $\sigma_H = 0.55$, $\gamma = 0.22$, $\tilde{b}_L = 0.065$. Since this is not intended as a quantitative exercise, we choose these particular values of $\sigma_L, \sigma_H, \gamma, \tilde{b}_L$ purely to make the qualitative features of equilibrium described in Proposition 2 easy to see. These properties of equilibrium do not depend qualitatively on the choice of parameters.

[24] This is the allocation that would be attained in an economy with a ZLB but without nominal wage rigidity.

very slight decline in capital, output, and real wages (panels (a), (b) and (e) respectively), the economy naturally remains at full employment throughout (panel (c)).

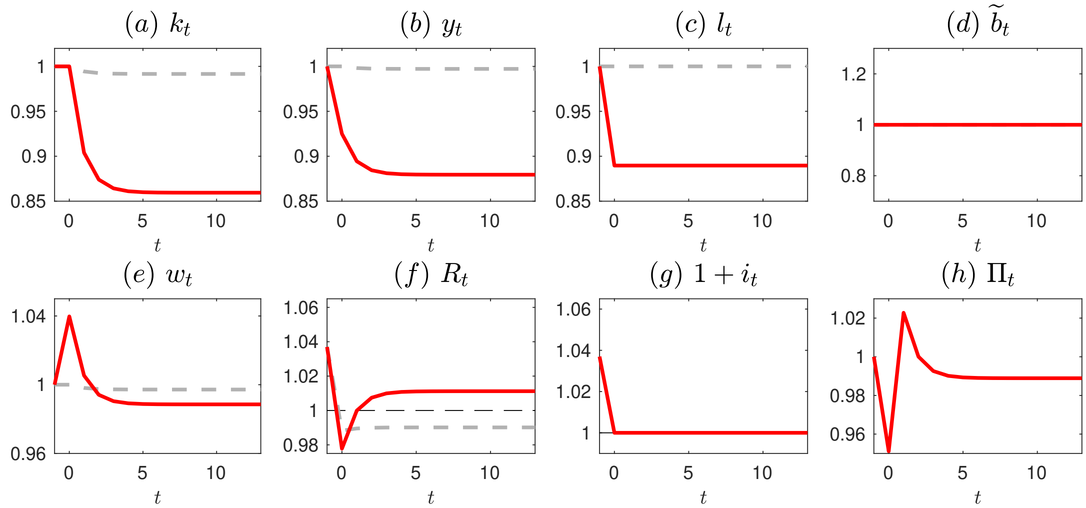


Figure 4. Risk-induced stagnation. Dashed lines denote the natural allocation following the permanent increase in σ while the solid lines denote the equilibrium trajectory of the economy with nominal rigidities.

In contrast, the solid red lines illustrate dynamics in the economy with nominal rigidities. The increase in risk, and the associated fall in employment, permanently reduce the aggregate saving rate, causing capital to decline to a lower steady state level (panel (a)). Real interest rates (panel (f)) fall on impact, as the spread between the safe rate and the expected MPK increases. As the capital stock declines, expected MPK rises while the spread remains wider, leading the real rate to increase to its new steady state level. Employment (panel (c)) falls to its new lower steady state level. The fall in capital and employment combine to create a sustained decline in output (panel (b)). Finally, panel (h) depicts inflation. The collapse in demand at date 0 causes a large fall in prices, pushing up real wages (panel (e)) and creating unemployment. Inflation then recovers somewhat before declining to its new steady state level. Intuitively, this economy requires lower interest rates early on in the transition to a new steady state, as the capital stock remains high and the MPK remains low. With the nominal rate stuck at zero (panel (g)), real rates can only be temporarily low if inflation is temporarily high. As the capital stock declines, the real interest rate rises somewhat, and inflation falls further.

Stagnant steady states To understand why nominal rigidities permit permanently high unemployment, it is useful to revisit our analysis of steady states. While the flexible wage economy was always at full employment, nominal wage rigidities allow the labor market to be in one of two regimes. In steady state, (21) becomes the long run Phillips curve:

$$l = \min\left\{\Pi^{\frac{1-\gamma}{\alpha\gamma}}, 1\right\} \tag{25}$$

Intuitively, since real wages are constant in steady state, positive steady state inflation ($\Pi \geq 1$) implies that nominal wages must be rising, effectively making wages flexible and ensuring full employment, $l = 1$. In contrast, with deflation ($\Pi < 1$), nominal wages must be falling. Since nominal wages are slow to adjust downwards, they cannot catch up with declining prices. Thus real wages exceed their market clearing level and there is unemployment in steady state. (25) defines an increasing relationship between inflation and employment in this regime whose slope depends on the degree of wage flexibility γ. When $\gamma = 1$ (perfect flexibility), the Phillips curve is vertical at full employment. When $\gamma = 0$ (perfect downward rigidity), the Phillips curve is inverse-L shaped and is horizontal at zero inflation ($\Pi = 1$). Thus, with $\gamma < 1$, in the deflation regime, inflation affects real allocations in the long run.

Our monetary policy rule (22) also implies two steady state regimes. Either the ZLB is slack ($i > 0$) and monetary policy achieves its target $\Pi l^{\psi(1-\alpha)} = 1$, or the ZLB binds ($i = 0$) and $\Pi l^{\psi(1-\alpha)} \leq 1$. In fact, the two regimes of the Phillips curve and monetary policy coincide in steady state: we either have $l = 1$ or $i = 0$ and $R = \Pi^{-1}$. In the ZLB regime, since $i = 0$ and $\Pi < 1$, the real interest rate $R > 1$: higher steady state unemployment generates more deflation and a higher real interest rate. Combining the monetary policy rule and the Phillips curve yields the following set-valued map which we refer to as the *'LM'* curve (Labor markets and Monetary policy), depicted by the solid red curve in the left panel of Figure 5:

$$R = \begin{cases} l^{-\frac{\alpha\gamma}{1-\gamma}} & \text{if } l < 1 \\ r & \text{for any } r \geq 1 \text{ if } l = 1 \end{cases} \tag{26}$$

Finally, evaluating (23)-(24) at steady state describes young households' investment and savings decisions, expressing k and R as functions of steady state employment (these are the same functions defined in (16)):

$$k = k\left(\tilde{b}l^{\alpha-1}, \sigma\right) l \qquad \text{and} \qquad R \;=\; R\left(\tilde{b}l^{\alpha-1}, \sigma\right) \tag{27}$$

These expressions define an increasing relationship between the capital stock and employment (upward sloping dashed black line in the right panel of Figure 5) and a decreasing relationship between interest rates and employment (downward sloping dashed blue curve in the left panel of Figure 5) respectively. Intuitively, higher employment raises labor income, increasing savings and steady state capital. Similarly, higher employment implies higher steady state capital and investment; for households to invest more in capital, rather than safe government debt, real interest rates must fall. Thus, the blue curves show the relation between l and R required to equate saving and investment in steady state. We refer to them as IS curves.

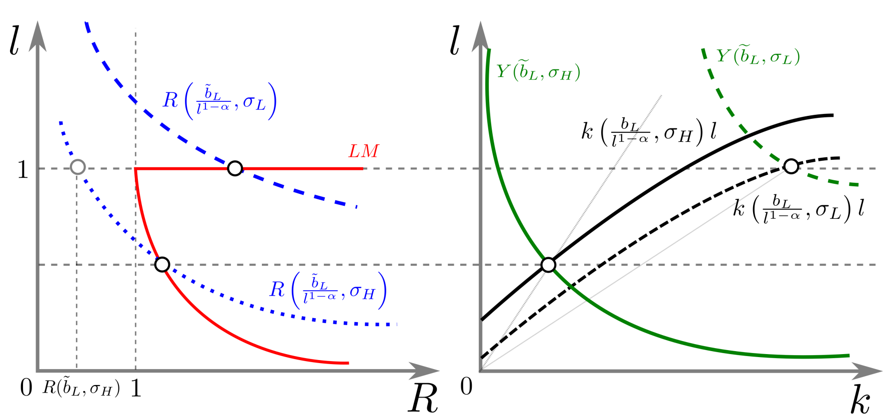


Figure 5. A permanent increase in σ keeping $\tilde{b}$ fixed

The intersection of the IS and LM curves determines steady state R and l. To understand how the

ZLB affects long-run outcomes, Figure 5 compares steady states with σ_L (in which the ZLB doesn't bind) and $\sigma_H > \sigma_L$ (in which it does). The dashed blue line in the left panel, denoting the IS curve with low risk, intersects the full employment part of the LM curve at $R > 1$. The right panel shows that full employment generates a high steady state capital stock. The green downward sloping curves on the right panel indicate isoquants of the aggregate production function, $Y = k^\alpha l^{1-\alpha}$. Full employment and a high capital stock produce relatively high output in this low risk steady state, shown by the dashed-green higher isoquant.

Higher risk (σ_H) shifts the IS curve left (to the dotted blue curve in the left panel), as savers substitute from riskier capital towards safe debt, so that a lower real rate is required for them to hold capital. Indeed, this IS curve intersects the dashed horizontal full employment line at $R < 1$, indicating that the steady state natural rate (the rate required to sustain full employment) is negative. Given the ZLB, the LM curve does not permit $R < 1$. Instead, the ZLB binds, and the IS and LM curves intersect at $l < 1$. Unemployment in turn generates persistent deflation, raising real rates further above the natural rate with the nominal rate stuck at zero. The economy enters a stagnant steady state. Permanent unemployment implies lower income for young savers, less investment, and lower capital (solid black line in the right panel). Lower capital and employment produce lower output (lower solid green isoquant in the right panel). Higher risk reduces the capital-labor ratio (gray lines passing through the the origin in the right panel).

4.2 How safe asset creation can restore full employment

An increase in government debt can offset an increase in risk and keep the natural rate of interest positive by satiating the demand for safe assets. Absent nominal rigidities, there was no reason to do so; but in the presence of nominal rigidities, a negative natural rate can cause a permanent recession, as shown above. As we now show, issuing more debt can prevent a recession.

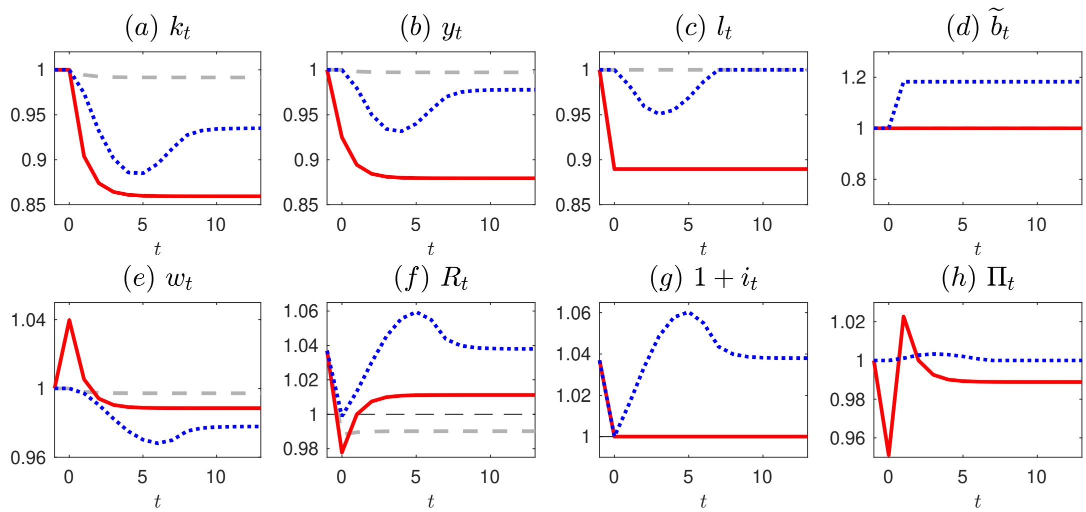


Figure 6. An increase in the supply of safe assets Dashed lines denote the natural allocation with no increase in safe assets. Solid red lines denote equilibrium with nominal rigidities and no increase in safe assets. Dotted blue lines denote equilibrium with nominal rigidities and an increase in safe assets.

The blue solid lines in Figure 6 depict the equilibrium when the fiscal authority raises $\tilde{b}_{t+1}$ permanently from $\tilde{b}_L$ to a higher level $\tilde{b}_H$ starting at date 0.[25] This increase in the supply of safe assets (panel (d)

[25] In Figure 6 we set $\tilde{b}_H = 0.077$. The precise value of $\tilde{b}_H$ is not important as long as it corresponds to a positive steady state natural rate of interest with high risk, i.e. $R(\tilde{b}_H, \sigma_H) > 1$. We set $\psi = 0.1$, which implies a relative weight on output

) accommodates the higher demand induced by the increase in risk, equilibrating asset markets without requiring the monetary authority to cut nominal rates below zero. With more safe assets in their portfolio, households are less averse to investing in capital;[26] at the same time the government rebates the proceeds from debt issuance to households, allowing them to spend more on consumption and capital. This mitigates the fall in aggregate spending, preventing prices from falling. With no fall in prices, the nominal wage rigidity does not bind. Consequently, the higher $\tilde{b}$ prevents the increase in risk from reducing employment (panel (c)) and output (panel (b)) on impact.

A permanently higher $\widetilde{b}$ causes a transition to a new steady state with a lower level of capital than before the increase in risk. However, this level of capital is still higher than the level that would obtain without an increase in debt, given that nominal wages are not fully flexible; increasing debt short-circuits the adverse feedback loop between unemployment and low investment. The economy experiences unemployment in the short-term since the gradual decrease in capital lowers the marginal product of labor and hence the market clearing real wage. Since wages are slow to adjust, this leads to unemployment in the short-term, but eventually the economy reaches full employment (panel (c)). In contrast, without an increase in debt, the economy faces permanently lower employment and investment. Thus, in the presence of nominal rigidities, increasing debt in a liquidity trap actually raises investment (panel (a)) and restore full employment.

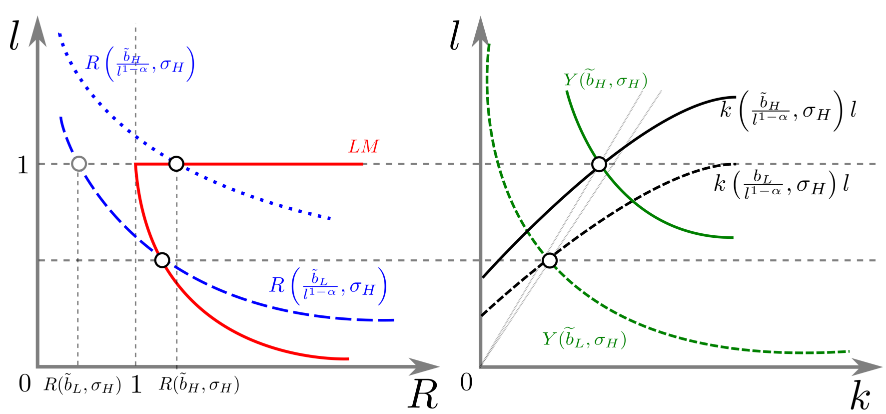


Figure 7. A permanent increase in $\tilde{b}$ in a high σ environment

The long-run effects of increasing $\widetilde{b}$ can also be understood using Figure 7. Higher debt satiates the demand for safe assets, reducing the risk premium, shifting the IS curve rightwards (from the dashed blue curve to the dotted blue curve in the left panel), and raising the natural rate (intersection of the IS curve and $l = 1$). A large enough increase in $\widetilde{b}$ pushes the natural rate above zero, allowing monetary policy to equate the real rate and the natural rate and achieve full employment. As the right panel shows, higher $\widetilde{b}$ increases capital relative to the stagnant steady state: higher employment raises the MPK, encouraging investment. (The dashed lines in the right panel depict the steady state without the increase in debt; the solid lines depict steady state with higher debt.)

While an increase in $\widetilde{b}$ raises investment, output and employment in a liquidity trap, it ultimately leaves the capital stock lower than before the increase in risk. As the right panel of Figure 7 shows, higher $\tilde{b}$ reduces steady state capital for any level of employment (moving from the upward sloping dashed black line

stabilization of 10 percent which is relatively standard. Again, the precise value of ψ does not affect the outcomes qualitatively.

[26]Recall that the households' demand for capital is increasing in $\tilde{b}$: As the consumption of the old has a lower covariance with the return on capital, capital becomes a more attractive option.

to the solid black line), reducing investment relative to the natural allocation with no additional safe assets. With lower capital, output falls below its level in the original steady state even though full employment has been restored, as the isoquants (downward sloping solid green curve) show. Indeed the new steady state has a lower capital-labor ratio, not just relative to the low risk steady state but also the stagnant steady state (gray lines in the right panel). This *capital shallowing* in turn reduces real wages and labor productivity (Y/l). In this sense, a recession induced by an increased demand for safe assets can continue to depress output, wages and labor productivity even when fiscal policy has restored full employment. Similarly, along the transition, increasing $\widetilde{b}$ yields higher output and investment relative to the equilibrium with nominal rigidities and low debt (compare dotted blue and solid red lines in panels (a) and (b) of Figure 6), but lower output and investment than in the low debt natural allocation (gray dashed line).

Our model predicts that at the ZLB, an increase in safe asset creation should be associated with a gradual return to full employment, but persistently lower investment, labor productivity and real wages. A growing literature argues that the U.S. has witnessed sluggish investment following the Great Recession and has proposed various explanations - declining competition (Gutierrez and Philippon, 2017), the rising importance of intangible assets (Crouzet and Eberly, 2018). Our model offers a different perspective. Even once the economy has returned to full employment, the higher supply of public safe assets can permanently depress investment. While higher debt raises the natural rate of interest above zero, negative real interest rates may be necessary to support high investment.

Despite these side effects on investment, if safe asset creation is the only tool available to a policy-maker, it should always be used to restore full employment. Consider the problem of a constrained planner who chooses $\tilde{b}$ to maximize steady state welfare, given the constraints imposed by nominal rigidities and monetary policy:

$$\max_{k,\tilde{b},l,R,\Pi,i} (1-\beta)\ln\left[(1-\alpha)k^{\alpha}l^{1-\alpha} - \tilde{b}k^{\alpha} - k\right] + \beta\mathbb{E}_z \ln\left[\alpha z k^{\alpha}l^{1-\alpha} + \tilde{b}k^{\alpha}\right] \tag{28}$$

subject to (27) which describes steady state capital labor ratio and real interest rates, the steady state Phillips curve (25) and the monetary policy rule which, in steady state, reduces to:

$$\left(\frac{\Pi}{\Pi^{\star}}\right) l^{(1-\alpha)\psi} \leq 1, \qquad 1+i = R\Pi \geq 1, \qquad i\left[\left(\frac{\Pi}{\Pi^{\star}}\right) l^{(1-\alpha)\psi} - 1\right] = 0$$

In the presence of nominal rigidities, it is constrained optimal to create enough safe assets to keep the natural rate nonnegative, as the Proposition below shows.

Proposition 3 (Constrained optimal allocation with nominal rigidities)**.** *Let $\tilde{b}_{real}(\sigma)$ be the choice of $\tilde{b}$, given σ, which maximizes steady state welfare in Proposition 1, and define $\tilde{b}_{zlb}(\sigma,\Pi^{\star})$ as the smallest level of $\tilde{b}$ such that the steady state features $i \geq 0$ and $l = 1$. Then the level of $\tilde{b}$ which solves (28) is:*

$$\tilde{b} = \max\left\{\tilde{b}_{zlb}(\sigma,\Pi^{\star}), \tilde{b}_{real}(\sigma)\right\}$$

In particular, if $\Pi^{\star} = 1$, then $\tilde{b} = \tilde{b}_{zlb}(\sigma,1) > \widetilde{b}_{real}(\sigma)$ whenever $\sigma > \underline{\sigma}$ (i.e. if the ZLB binds with $\tilde{b} = 0$). Consequently, starting from a level of safe assets $\widetilde{b}$ at which the ZLB binds $R(\widetilde{b},\sigma) < 1$, increasing safe assets to a level $\widetilde{b}_{zlb}(\sigma,1) > \widetilde{b}$ increases steady state welfare.

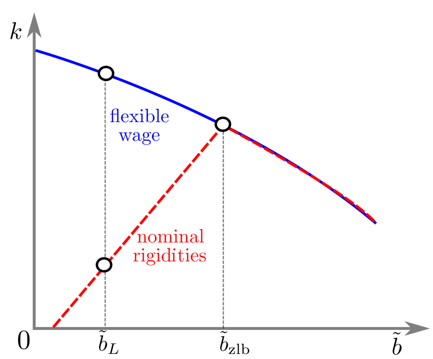
Proof. See Appendix I. □

Absent nominal rigidities, $\tilde{b} = 0$ was optimal because increasing $\tilde{b}$ would reduce steady state capital. In an economy with nominal rigidities, since increasing $\tilde{b}$ up to $\tilde{b}_{\text{zlb}}(\sigma, \Pi^\star)$ actually *increases* capital relative to the stagnant steady state, this reason for abstaining from safe asset production no longer applies. Of course, once the economy has reached full employment, a further increase in $\tilde{b}$ would only crowd out capital, reducing welfare. Figure 8a illustrates the trade-off between steady state k and $\tilde{b}$ when $\sigma > \underline{\sigma}$, i.e. we would have $R(0, \sigma_H) < 1$ absent safe assets. The blue line illustrates this relation in the natural allocation, which is always decreasing. Absent nominal rigidities, increasing safe assets from $\tilde{b}_L$ to $\tilde{b}_H$ always decreases steady state capital - such an increase is generally undesirable and tends to reduce welfare, as shown in the solid blue curve in Figure 8b.[27] In contrast the dashed-red curve in Figure 8a depicts the same relationship, but with nominal rigidities. Now refraining from additional safe asset production results in unemployment, lowering steady state capital. Increasing $\tilde{b}$ up to $\tilde{b}_{\text{zlb}}$ increases capital, and thus also increases welfare (shown by the dashed red line in Figure 8b). Beyond this point, the ZLB no longer binds and the economy behaves as in the natural allocation. Thus, it is optimal to increase $\tilde{b}$ to $\tilde{b}_{\text{zlb}}$ but no more.

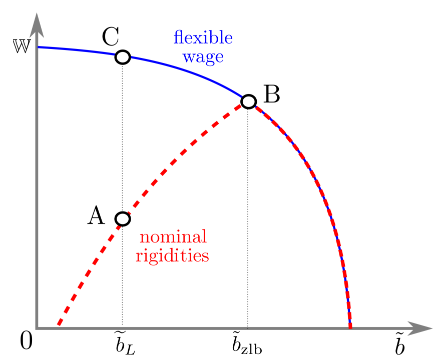
A binding ZLB changes the net benefit of safe asset creation from (19) in the natural allocataion to:

$$\left[-\left(\frac{R-1}{R}\right) + (1-\alpha)\left[\mathbb{E}_z R^k(z) - 1\right]\frac{dk}{db} + \left(\frac{1-\alpha}{\alpha}\right)\left[(1-\alpha)\,\mathbb{E}_z R^k(z) + \alpha\right]\frac{k}{l}\frac{dl}{db}\right]\left(\frac{1-\beta}{c^Y}\right) \tag{29}$$

Absent the ZLB, the direct benefit of safe asset creation (first term in the square brackets in (29)) is balanced by the indirect cost of crowding out capital (second term in (29)). When the ZLB binds, higher debt increases rather than reduces steady state capital $\left(\frac{dk}{db} > 0\right)$, and this indirect effect becomes an additional *benefit* of safe asset creation. Higher debt also increases employment $\left(\frac{dl}{db} > 0\right)$ as long as the ZLB is binding, further increasing the benefit of safe asset creation (third term in (29)). Thus, higher debt unambiguously increases welfare at the ZLB (moving from point A to B in Figure 8b) even though it results in lower welfare relative to a flexible wage allocation with lower debt (point C in Figure 8b).



(a) steady state capital



(b) steady state welfare when $\sigma < \overline{\sigma}$

Figure 8. Steady state capital and welfare as functions of $\tilde{b}$ in an environment with $\sigma > \underline{\sigma}$

A high-risk economy needs negative real rates to sustain high investment, as in the optimal natural allocation. With nominal rigidities and a zero long run inflation target, negative rates are not possible. Thus an economy with a negative natural rate experiences a recession, as monetary policy is constrained

[27] As mentioned in Section 3, if risk is sufficiently high, $\sigma > \overline{\sigma}$, the optimal natural allocation features positive government debt but not enough to make $R > 1$. That is $\tilde{b}_{real}(\sigma)$ is positive but less than $\tilde{b}_{zlb}(\sigma, 1)$.

by the ZLB. Issuing public debt satiates the demand for safe assets and raises the natural rate of interest, relaxing the ZLB. But this does not change the fact that a risky economy *requires negative real rates* to sustain high *investment*. The same increase in debt which restores full employment crowds out investment, selecting a different, *sub-optimal* natural allocation. Despite these costs, safe asset creation may still be desirable if other policy instruments are available. Next, we show that raising the inflation target restores full employment without crowding out investment, and may be preferable to increasing debt.

4.3 Raising the Inflation Target

Suppose that at date 0, in response to the increase in σ, fiscal policy keeps $\tilde{b}$ unchanged at $\tilde{b}_L$ but monetary policy raises the inflation target to $\Pi^\star > 1$. This changes the steady state LM relation (21) to:

$$R = \begin{cases} \frac{l^{-\frac{\alpha\gamma}{1-\gamma}}}{\Pi^\star} & \text{if } l < 1 \\ r & \text{for any } r \geq \frac{1}{\Pi^\star} \text{ if } l = 1 \end{cases} \tag{30}$$

With a higher $\Pi^\star$, monetary policy does not raise rates even when the economy experiences full employment and modestly positive inflation. This allows for a steady state with full employment and negative real interest rates, which would not be possible if monetary policy targeted zero inflation. In other words, as the left panel of Figure 9 shows, raising $\Pi^\star$ from 1 shifts the LM curve leftwards, moving from the dashed to the dotted red curve. This closes the gap between a lower natural rate and a higher prevailing rate of interest by reducing the prevailing rate, rather than by increasing the natural rate (as in Figure 7). A large enough increase in $\Pi^\star$ maintains full employment even after the increase in risk and a fall in the natural rate. This allows the shifted LM curve to intersect the IS curve (solid blue curve) at full employment and negative real rates.

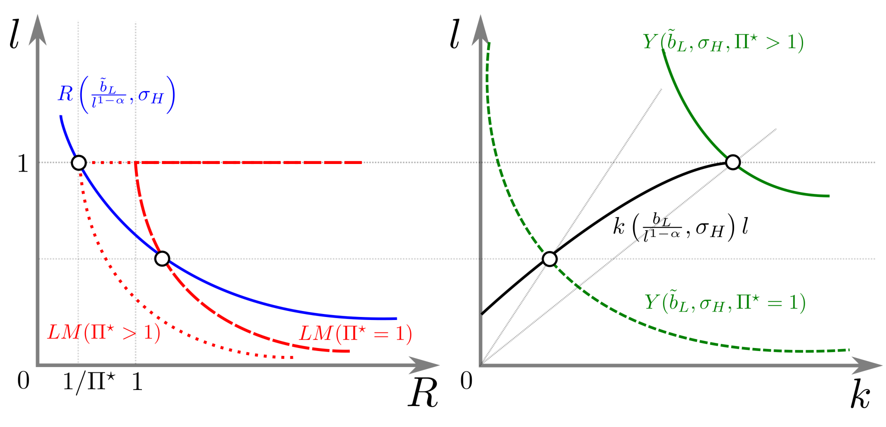


Figure 9. An increase in $\Pi^\star$ in a high σ environment.

The right panel of Figure 9 shows that by attaining full employment, higher $\Pi^\star$ allows both higher output and a higher capital-labor ratio relative to zero target inflation and no increase in safe assets. Unlike an increase in the supply of safe assets (see Figure 7), higher $\Pi^\star$ does not crowd out investment. Graphically, the black curve in the right panel of Figure 9, depicting the relation between steady state capital and employment, does not shift leftwards as it did in Figure 7. Thus higher target inflation permits higher output, capital-labor ratio and labor productivity relative to an increase in safe assets.

Higher $\Pi^\star$ also lead to smoother transitions. As before, the dotted blue and solid red lines in figure 10 depict dynamics with and without an increase in safe assets, respectively, and with $\Pi^\star = 1$. The dot-dashed black lines depict transitional dynamics with no increase in safe assets but an increase in the inflation target from $\Pi^\star = 1$ to $\Pi^\star = 1.02$ starting from date $t = 0$. With higher target inflation, real rates can fall persistently below zero without the ZLB binding. Low real rates keep investment high, and the decline in capital is smaller than in the case with safe asset creation (dotted blue line in panel (a)), not to mention the case without safe asset creation and $\Pi^\star = 1$ (solid red line in panel (a)). Indeed, the black line in panel (a) closely mimics the trajectory of the capital stock in the natural allocation with no increase in safe assets. The only difference is that in the short run a reduction in capital requires a small decline in real wages, which causes temporary unemployment (panel (c)) and above-target inflation (panel (d)). However, this is temporary and long run outcomes coincide with the natural allocation.

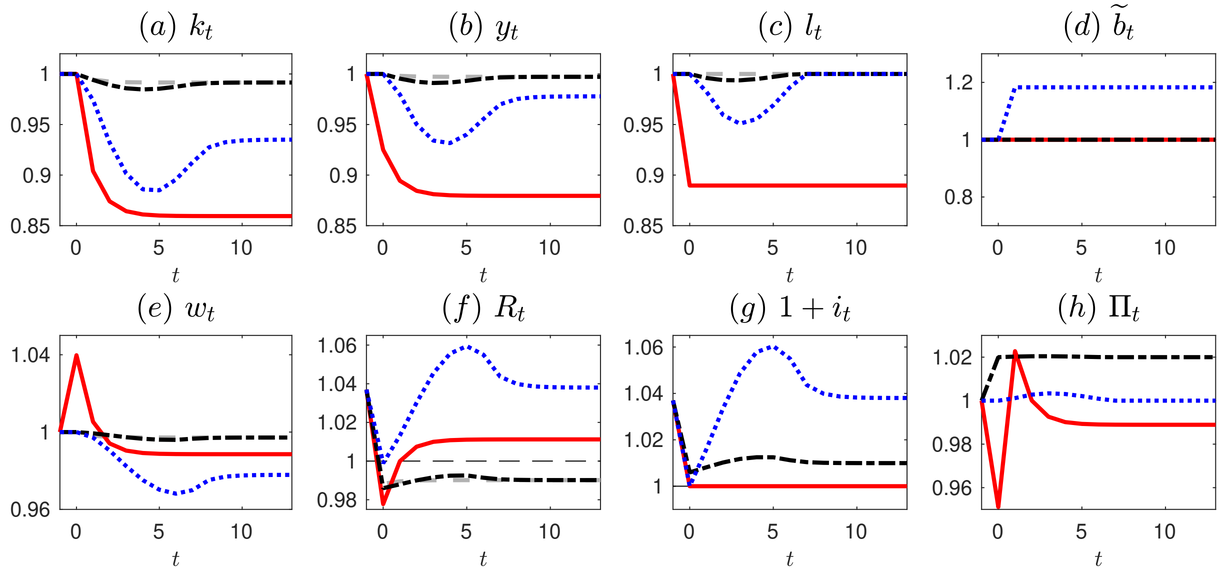


Figure 10. An increase in $\Pi^\star$. Dashed lines denote equilibrium with no increase in safe assets, absent nominal rigidities . Solid red lines denote equilibrium with nominal rigidities and no increase in safe assets. Dotted blue lines denote equilibrium with an increase in safe assets and no change in the inflation target. Dot-dashed black lines denote equilibrium with an increase in the inflation target and no increase in safe assets.

Traditionally, economists have argued that monetary policy should seek to replicate natural allocations (Goodfriend and King, 1997). Our economy has many natural allocations indexed by $\tilde{b}$, which is a *choice variable* of the fiscal authority. $\tilde{b} = 0$ selects the optimal natural allocation,[28] even if this involves negative real rates. Without higher target inflation, it is not possible to replicate this optimal natural allocation (or more generally the pre-recession level $\tilde{b}_L$). Safe asset creation shifts the goal posts, presenting monetary policy with the easier task of implementing a different, suboptimal natural allocation. But to replicate the *optimal* natural allocation requires negative real rates, which are essential to sustain high investment in a high risk economy. Higher inflation is the only way to deliver this given the ZLB. In other words, even if it is desirable to close the gap between a negative natural rate and a prevailing real rate stuck above zero, there are two ways to do this. Safe asset creation raises the natural rate to meet the higher prevailing rate, which crowds out capital – which reduces welfare, since our economy is dynamically efficient. A higher inflation target instead reduces the prevailing rate to meet the lower natural rate in the optimal natural

[28] Provided that $\sigma < \bar{\sigma}$.

allocation, sustaining high investment. The distinction between these two types of policies is immaterial in the analysis of Caballero and Farhi (2016), for whom higher debt and higher inflation targets are equally desirable: they consider economies without capital in which it does not matter whether the natural and prevailing real rates are equated at a high or low level.[29]

To be clear: creating safe assets is better than doing nothing. But it is inferior to a policy which refrains from safe asset creation and instead implements negative real interest rates, e.g. through higher inflation targets. Our model does not permit a full cost-benefit analysis of higher inflation targets, since it abstracts from the costs associated with higher steady state inflation (Coibion et al., 2016). In an environment where trend inflation is costly, some combination of higher debt and a higher inflation target would be optimal, but it would never be optimal to issue enough debt to raise the real natural rate above zero.

5 Conclusion

We presented a model to evaluate the costs and benefits of increasing government debt when an increase in the demand for safe assets causes the ZLB to bind. Increasing debt prevents the natural rate from falling below zero, relaxing the ZLB and preventing a permanent economic slump. However, this comes at the cost of permanently lower investment, which reduces welfare because the economy is dynamically efficient even when the natural rate is negative. Rather than increasing debt, it may be preferable to raise the inflation target, permitting negative real rates; this sustains high investment while preventing unemployment.

Our analysis also provides a new perspective on the idea that deficit spending is doubly desirable in ZLB episodes, because higher deficits reduce unemployment in the short run, while negative real rates make it a good time for the government to borrow. In our model, higher deficits do prevent unemployment, and actually increase investment. This is preferable to tight fiscal policy and a low inflation target. But while deficit spending implements *a* natural allocation, this is not the *optimal* natural allocation. Raising the inflation target may be preferable to either deficit spending or tight fiscal policy and a low inflation target.

While our paper is not quantitative, the scenario we have described is similar to the experience of many advanced economies following the Great Recession, which saw a large increase in publicly issued safe assets (government debt held by the public and central bank reserves). Even after returning to full employment, output, investment and labor productivity remained persistently below pre-crisis trends. Our analysis suggests that these outcomes might be the side effects of an increase in safe asset creation.

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[29]In a riskless economy with capital and negative interest rates, increasing government debt would actually be preferable to higher inflation targets since it would crowd out capital which is welfare improving. The reason higher inflation targets may be preferable to an increase in debt is that our economy features not just capital, but risk – implying that the economy can be dynamically efficient even when the real natural rate is negative.

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Appendix

A Household's Optimal Choices

Using equations (1)-(2), the objective function of the households can be written as:

$$\max_{k_{t+1},b_{t+1}} (1-\beta)\ln\left[\omega_t l_t + T_t - \frac{1}{R_t}b_{t+1} - k_{t+1}\right] + \beta\mathbb{E}_z \ln\left[R^k_{t+1}(z)k_{t+1} + b_{t+1}\right]$$

where $\omega_t = \frac{W_t}{P_t}$ and $b_{t+1} = \frac{B_{t+1}}{P_{t+1}}$ and $R_t = \frac{\Pi_t}{1+i_t}$. The first order conditions w.r.t k_{t+1} and b_{t+1} can be written as:

$$\frac{1-\beta}{\omega_t l_t + T_t - \frac{b_{t+1}}{R_t} - k_{t+1}} = \beta\mathbb{E}_z\left[\frac{R^k_{t+1}}{R^k_{t+1}(z)k_{t+1} + b_{t+1}}\right] \tag{31}$$

$$\frac{1-\beta}{\omega_t l_t + T_t - \frac{b_{t+1}}{R_t} - k_{t+1}} = \beta\mathbb{E}_z\left[\frac{R_t}{R^k_{t+1}(z)k_{t+1} + b_{t+1}}\right] \tag{32}$$

Next multiply equation (31) by k_{t+1}, (32) by $\frac{b_{t+1}}{R_t}$ and add them up:

$$\frac{k_{t+1}+\frac{b_{t+1}}{R_t}}{\omega_t l_t + T_t - \frac{b_{t+1}}{R_t} - k_{t+1}} = \frac{\beta}{1-\beta} \tag{33}$$

which can be rearranged to yield:

$$k_{t+1} + \frac{b_{t+1}}{R_t} = \beta\left[\omega_t l_t + T_t\right] \tag{34}$$

i.e. the young household save a fraction β of its labor income net of transfers. Using the budget constraint, it is straightforward to see that

$$c^Y_{t+1} = (1-\beta)\left[\omega_t l_t + T_t\right]$$

Using these equations, we can re-write the objective as:

$$\max_{\eta_t}(1-\beta)\ln\Big[(1-\beta)(\omega_t l_t + T_t)\Big] + \beta\ln\Big[(\omega_t l_t + T_t)\Big] + \beta\mathbb{E}_z\ln\Big[\eta_t R^k_{t+1}(z) + (1-\eta_t)R_t\Big]$$

where we define the portfolio share of capital as η_t as $\frac{k_{t+1}}{k_{t+1}+\frac{b_{t+1}}{R_t}}$. Notice that only the last term of the expression depends on η_t. Thus, the choice of η_t can be seen as choosing portfolio weights to maximize risk-adjusted returns:

$$\eta = \arg\max_{\eta}\mathbb{E}_z\ln\Big[\eta_t R^k_{t+1}(z) + (1-\eta_t)R_t\Big]$$

The optimal choice of η_t can then be written as:

$$\mathbb{E}_z\Big[\frac{R^k_{t+1}(z) - R_t}{\eta_t R^k_{t+1}(z) + (1-\eta_t)R_t}\Big] = 0$$

Notice that the numerator of the expression above is the return earned by capital in excess of bonds and the denominator is just the return on a portfolio with share of capital being η_t. To derive equation 6, use the capital Euler equation of a household

$$\frac{1-\beta}{c^Y_t} = \beta\mathbb{E}_z\frac{R^k_{t+1}(z)}{c^O_{t+1}(z)}$$

where $c^O_{t+1}(z) = R^k_{t+1}(z)k_{t+1} + b_{t+1}$. Using the fact that $c^Y_t = \frac{1-\beta}{\beta}(k_{t+1} + b_{t+1}/R_t)$ and multiplying both sides of the Euler equation by k_{t+1} yields the expression (6) in the main text.

B Deriving an Expression for the Real Interest Rate

Using equations (4)-(5) we know that:

$$\frac{1-\eta_t}{\eta_t} = \frac{b_{t+1}}{R_t k_{t+1}} = \frac{\tilde{b}_{t+1}}{R_t}k^{\alpha-1}_{t+1}$$

where we used the definition of $\tilde{b}$ to go from the first to the second equality. Substituting out η_t using (12) and rearranging:

$$R_t = \frac{\mathbb{E}_z\left[\frac{z}{\alpha z+\tilde{b}_{t+1}}\right]}{\mathbb{E}_z\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]}\alpha k_{t+1}^{\alpha-1} = \frac{\mathbb{E}_z\left[\frac{z}{\alpha z+\tilde{b}_{t+1}}\right]}{\mathbb{E}_z\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]}\mathbb{E}_z R_{t+1}^k(z)$$

Rearranging we have equation (13). Notice that we can also write the expression for R_t as:

$$R_t = \left(\mathbb{E}_t\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]^{-1} - \tilde{b}_{t+1}\right)k_{t+1}^{\alpha-1}$$

Then since $\mathbb{E}_t\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]$ is increasing in σ (from Jensen's inequality), the whole expression is decreasing and thus, $\frac{\partial R_t}{\partial \sigma} < 0$. Notice also that the inverse of the spread can be written as:

$$\frac{R_t}{\mathbb{E}_z R_{t+1}^k(z)} = \frac{1}{\alpha}\frac{1-\mathbb{E}_z\left[\frac{\tilde{b}_{t+1}}{\alpha z+\tilde{b}_{t+1}}\right]}{\mathbb{E}_z\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]} = \frac{1}{\alpha}\left[\mathbb{E}_z\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]^{-1} - \tilde{b}_{t+1}\right]$$

Next, from Jensen's inequality, we know that:

$$\frac{\partial\left(\frac{R_t}{\mathbb{R}_{t+1}^k(z)}\right)}{\partial \tilde{b}_{t+1}} = \frac{\mathbb{E}\left[\left(\frac{1}{\alpha z+\tilde{b}_{t+1}}\right)^2\right] - \left(\mathbb{E}\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]\right)^2}{\mathbb{E}\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]\mathbb{E}\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]} > 0$$

C Model with trend productivity and population growth

In this Appendix, we extend the model to include trend population growth (at rate n) and technology growth (at rate g) into our risky economy. Specifically, we show that under our maintained assumptions, just like an increase in σ, a fall in g or n can drive the natural rate below zero in this economy. Furthermore, as in our baseline model, we show that if a fall in g or n or an increase in σ push the natural rate below zero, the economy remains dynamically efficient in the sense that an increase in steady state capital per effective worker increases steady state welfare. Thus, the tradeoffs remain broadly similar to those in the baseline model.

To include population and technology growth into the model, we assume that the production function of each old agent is now given by:

$$Y_t(z) = (zk_t)^\alpha (A_t L_t)^{1-\alpha}$$

where A_t denotes the level of labor augmenting productivity growth while L_t denotes the size of the cohort born at date t. We assume that labor productivity A_t grows at a constant rate $g > 0 :\ A_t = e^{gt}A_0$ and that population grows at a constant rate $n :\ L_t = L_0 e^{nt}$.

Define $k_t = \frac{K_t}{A_t L_t}$ and $b_t = \frac{B_t}{A_t L_t}$ as capital per effective worker and bonds per effective worker. Then the steady state capital demand and supply equations can be written as:

$$\frac{1-\beta}{R}\widetilde{b} = \beta e^{-(g+n)}\left(1-\alpha-\widetilde{b}\right) - k^{1-\alpha}$$

$$\alpha k^{\alpha-1} = g(\widetilde{b}, \sigma)R$$

Notice that the capital supply equation is the same as in the baseline with $g = n = 0$ and the capital demand equation is unchanged. Here $\widetilde{b}$ is defined as debt per effective worker as a fraction of GDP. With $\widetilde{b} = 0$, the steady state real interest rate is given by:

$$R\,|_{\widetilde{b}=0} = \frac{\alpha}{\beta(1-\alpha)} e^{g+n-\sigma^2} \tag{35}$$

As in the baseline, Assumption 1 still ensures that the riskless economy is dynamically efficient in the sense that $\mathbb{E}_z R^k(z) = \alpha k^{\alpha-1} = \frac{\alpha}{\beta(1-\alpha)} e^{g+n} > e^{g+n}$. As can be seen, a fall in g or n can push the natural rate below zero provided that the level of risk is positive ($\sigma > 0$).

Next, welfare along a steady state balanced growth path can be written as:

$$\mathbb{W} = (1-\beta)\ln\left[(1-\alpha)\,k^{\alpha} - \widetilde{b}k^{\alpha} - ke^{g+n}\right] + \beta\mathbb{E}\ln\left[\left(\alpha z + \widetilde{b}\right)k^{\alpha}\right] + \ln A_t + \beta(g+n)$$

Notice that this is the same expression as in Section 3 with $g = n = 0$ and $A_0 = 1$. As in Section 3, as long as Assumption 1 is satisfied, welfare is increasing in the level of capital per effective worker (evaluating welfare at $b = 0$):

$$\left.\frac{d\mathbb{W}}{dk}\right|_{b=0} = \left[1 - \frac{\beta\,(1-\alpha)}{\alpha}\right]\frac{\alpha}{k} > 0 \qquad \because \text{Assumption } 1 \tag{36}$$

Note that this expression does not depend on g,n or σ. Thus, it follows that even if a fall in g or n (or an increase in σ) drives the natural rate below zero, the economy remains dynamically efficient in the sense that $\frac{d\mathbb{W}}{dk} > 0$.

D Proof of Lemma 4

The FOC for the choice of $\tilde{b}$ can be written as:

$$-\frac{1-\beta}{(1-\alpha-\tilde{b})k^{\alpha} - k} + \beta\mathbb{E}_z\left[\frac{1}{\left(\alpha z + \tilde{b}\right)k^{\alpha}}\right] \leq 0 \qquad \text{and} \qquad \tilde{b} \geq 0 \tag{37}$$

with at least one of the conditions holding with a strict equality. Also, note that the young household's bond Euler equation in equilibrium can be written as:

$$-\frac{1-\beta}{(1-\alpha-\tilde{b})k^{\alpha} - k} + \beta R\mathbb{E}_z\left[\frac{1}{\left(\alpha z + \tilde{b}\right)k^{\alpha}}\right] = 0$$

Combining this equation with (37), we can write optimality as: $R \geq 1$, $\tilde{b} \geq 0$ and $(R-1)\tilde{b} = 0$.

E Proof of Proposition 1

The problem of the steady state planner can be written as:

$$\mathcal{L} = \max_{k,\tilde{b}\geq 0}(1-\beta)\ln\left[(1-\alpha-\tilde{b})k^{\alpha}-k\right]+\beta\mathbb{E}_z\ln\left[(\alpha z+\tilde{b})k^{\alpha}\right]-\lambda^{ss}\left(k-s(\tilde{b},\tilde{b},\sigma)^{\frac{1}{1-\alpha}}\right)$$

The FOC for k can be written as:

$$\frac{\alpha}{k}-\frac{(1-\beta)(1-\alpha)}{(1-\alpha-\tilde{b})k^{\alpha}-k}-\lambda^{ss}=0 \tag{38}$$

The FOC for $\tilde{b}$ can be written as:

$$\frac{-(1-\beta)}{(1-\alpha-\tilde{b})k^{\alpha}-k}+\beta\mathbb{E}_z\left[\frac{1}{\left(\alpha z+\tilde{b}\right)k^{\alpha}}\right]+\frac{\lambda^{ss}s(\tilde{b},\tilde{b},\sigma)^{\frac{\alpha}{1-\alpha}}}{1-\alpha}\left[s_1(\tilde{b},\tilde{b},\sigma)+s_2(\tilde{b},\tilde{b},\sigma)\right] \;\leq\; 0 \tag{39}$$

$$\tilde{b} \;\geq\; 0 \tag{40}$$

To show that $\lambda^{ss}>0$

Proof of (i) The objective function can also be written in terms of (k,b):

$$\mathbb{W}(k,b)=(1-\beta)\ln\left[(1-\alpha)\,k^{\alpha}-b-k\right]+\beta\mathbb{E}\ln\left[\alpha z k^{\alpha}+b\right]$$

It is straightforward to see that $\mathbb{W}$ is concave in (k,b). Suppose $\sigma<\underline{\sigma}$ and evaluate the derivative of $\mathbb{W}$ at $(k_{\max},0)$ where $k_{\max}=s(0,0,\sigma)^{\frac{1}{1-\alpha}}$:

$$\begin{aligned}\frac{\partial\mathbb{W}(k_{\max},0)}{\partial k} &= (1-\beta)\frac{\alpha(1-\alpha)k_{\max}^{\alpha-1}-1}{(1-\alpha)k_{\max}^{\alpha}-k_{\max}}+\beta\frac{\alpha}{k_{\max}}\\ &= \frac{\alpha}{k_{\max}}\left[1-\frac{\beta(1-\alpha)}{\alpha}\right]>0\end{aligned} \tag{41}$$

where the last inequality stems from Assumption 1. Similarly,

$$\begin{aligned}\frac{\partial\mathbb{W}(k_{\max},0)}{\partial b} &= -(1-\beta)\frac{1}{(1-\alpha)(1-\beta)k_{\max}^{\alpha}}+\beta\mathbb{E}_z\left[\frac{1}{\alpha z k_{\max}^{\alpha}}\right]\\ &= -\frac{1}{(1-\alpha)k_{\max}^{\alpha}}\left[1-\frac{\beta(1-\alpha)}{\alpha}e^{\sigma^2}\right]<0\end{aligned} \tag{42}$$

where the last inequality holds since $\sigma<\underline{\sigma}$.

Next, take any feasible allocation where $b>0$: it must feature $k<k_{\max}$. Since $\mathbb{W}(k,b)$ is concave,we have:

$$\begin{aligned}\mathbb{W}(k,b) &\leq \mathbb{W}\left(k_{\max},0\right)+\frac{\partial\mathbb{W}(k_{\max},0)}{\partial k}(k-k_{\max})+\frac{\partial\mathbb{W}(k_{\max},0)}{\partial b}b\\ &< \mathbb{W}(k_{\max},0)\end{aligned}$$

Thus, $(k_{\max},0)$ must be optimal. Since $\sigma<\underline{\sigma}$ and $b=0$, we know that $R(0,\sigma)>1$.

Proof of (ii) Substituting the implementability constraint into $\mathbb{W}(k, b)$, we have

$$\mathbf{W}(\tilde{b}, \varepsilon) := \mathbb{W}\left(s(\tilde{b}, \tilde{b}, \underline{\sigma} + \varepsilon)^{\frac{1}{1-\alpha}}, \tilde{b}s(\tilde{b}, \tilde{b}, \underline{\sigma} + \varepsilon)^{\frac{\alpha}{1-\alpha}}\right)$$

for $\varepsilon > 0$. In order for $\tilde{b} > 0$ to be optimal given ε, we need $\mathbf{W}_b(\tilde{b}, \varepsilon) = 0$ and $\mathbf{W}(\tilde{b}, \varepsilon) \geq \mathbf{W}(0, \varepsilon)$. It is straightforward to show that there exists a function $\tilde{\mathbf{b}}(\varepsilon)$ such that for $\tilde{b} > \tilde{\mathbf{b}}(\varepsilon)$, $\mathbf{W}(\tilde{b}, \varepsilon) < \mathbf{W}(0, \varepsilon)$. Further, $\tilde{\mathbf{b}}(\varepsilon) \to 0$ as $\varepsilon \to 0$. Next, note that $\mathbf{W}_b(\tilde{b}, \varepsilon)$ is a continuous function and is strictly negative at $(0, 0)$. Thus, there exists (γ, δ) such that $\tilde{b} \in (0, \gamma)$, $\varepsilon \in (0, \delta)$ implies $\mathbf{W}_b(\tilde{b}, \varepsilon) < 0.5\mathbf{W}_b(0, 0) < 0$. Choose $\varepsilon_1 < \delta$ such that $\tilde{\mathbf{b}}(\varepsilon_1) < \gamma$. For all $\varepsilon \in (0, \varepsilon_1)$, we have $\mathbf{W}_b(\tilde{b}, \varepsilon) < 0$ for all $\tilde{b} \in (0, \tilde{\mathbf{b}}(\varepsilon))$. Thus, there are no interior optimum and $\tilde{b} = 0$ must be optimal in an open interval around $\underline{\sigma}$. Since $\widetilde{b} > 0$ whenever $\sigma > \underline{\sigma}$ in the unconstrained problem, it follows that the constrained planner chooses strictly less safe assets than in the unconstrained problem.

Proof of (iii) First, we show that for σ sufficiently large, the following expression is positive

$$\begin{aligned} d\mathbb{W}(k_{\max}, 0) &= \frac{\partial \mathbb{W}(k_{\max}, 0)}{\partial k} \frac{s(0, 0, \sigma)^{\frac{\alpha}{1-\alpha}}}{1-\alpha} \Big[s_1(0, 0, \sigma) + s_2(0, 0, \sigma)\Big] + \frac{\partial \mathbb{W}(k_{\max}, 0)}{\partial \tilde{b}} \\ &= -\frac{1 - \beta\,(1-\alpha)}{(1-\alpha)^2} + \left[\frac{\beta - \alpha}{\beta(1-\alpha)} + 1 - \beta\right] \frac{\beta}{\alpha} e^{\sigma^2} \end{aligned}$$

For large enough σ the second term overwhelms the first term making $d\mathbb{W}(k_{\max}, 0) > 0$ if $\alpha < \beta$. In this case, there exists a finite $\bar{\sigma}$ such that as long as $\sigma > \bar{\sigma}$, it is optimal to create safe assets. If however α is large relative to β, and $\frac{\beta-\alpha}{\beta(1-\alpha)} + 1 - \beta < 0$, then it may never be optimal to create safe assets for any level of σ since crowding out always dominates the benefits from insurance.

It remains to show that at the optimum whenever $\tilde{b} > 0$, $R < 1$. First, we show that we can never have an interior optimum with $\mathbb{W}_k \leq 0$ and $\mathbb{W}_b < 0$. Consider any point $(k_0, \tilde{b}_0)$ with $b_0 > 0$ s.t. $\mathbb{W}_k(k_0, \tilde{b}_0) \leq 0$ and $\mathbb{W}_b(k_0, \tilde{b}_0) < 0$. For any $\varepsilon > 0$, define $k_\varepsilon = s(\tilde{b}_0 - \varepsilon, \tilde{b}_0 - \varepsilon, \sigma) < k_0$ as the steady state level of capital for $\tilde{b}_0 - \varepsilon$. The gain in welfare from decreasing $\tilde{b}$ by ε is approximately $\mathbb{W}_k(k_0, \tilde{b}_0)(k_\varepsilon - k_0) + \mathbb{W}_b(k_0, \tilde{b}_0)\varepsilon$. For small ε, this gain is positive since $\mathbb{W}_k \leq 0$, $k_\varepsilon < k_0$ and $\mathbb{W}_b > 0$. So the initial point cannot be optimal. By a similar argument, we cannot have both $\mathbb{W}_b \leq 0$ and $\mathbb{W}_k < 0$ at an optimum. Finally, since $\mathbb{W}$ is concave and attains its maximum at $\tilde{b} = 0$, $k > k_{max}$, we cannot have $\mathbb{W}_k = \mathbb{W}_b = 0$ at any feasible point.

Take any interior optimal point. The first order necessary condition for optimality is

$$\mathbb{W}_b + \mathbb{W}_k \frac{\partial k}{\partial b} = 0$$

If $\mathbb{W}_b \leq 0$ at an optimum, then by the above arguments we must have $\mathbb{W}_k > 0$, which contradicts the optimality condition. So at any interior optimum, we must have $\mathbb{W}_b > 0$, which, again using the household's Euler equation for bonds, implies that $R < 1$.

Finally, we show that the optimal $\widetilde{b}$ is higher in the unconstrained problem than in the constrained problem. It is useful to define $x = k^{1-\alpha}$. The objective function becomes

$$\mathbb{W} = (1-\beta)\ln\left[1 - \alpha - \widetilde{b} - x\right] + \beta\mathbb{E}_z \ln\left[\alpha z + \widetilde{b}\right] + \frac{\alpha}{1-\alpha}\ln x$$

The unconstrained planner maximizes this with respect to x and $\widetilde{b}$, yielding the first order necessary conditions for an interior optimum

$$\begin{aligned}\frac{d\mathbb{W}}{dx} &= -(1-\beta)\frac{1}{1-\alpha-\widetilde{b}-x}+\frac{\alpha}{1-\alpha}\frac{1}{x}=0\\ \frac{d\mathbb{W}}{d\widetilde{b}} &= -(1-\beta)\frac{1}{1-\alpha-\widetilde{b}-x}+\beta\mathbb{E}_z\frac{1}{\alpha z+\widetilde{b}}=0\end{aligned}$$

Using the first condition to solve for x, we can rewrite the second as

$$\frac{d\mathbb{W}}{d\widetilde{b}}=-\left[1-\beta+\frac{\alpha}{(1-\alpha)}\right]\frac{1}{1-\alpha-\widetilde{b}}+\beta\mathbb{E}_z\frac{1}{\alpha z+\widetilde{b}}:=\mathcal{D}\left(\widetilde{b}\right)=0$$

where $\mathcal{D}\left(\widetilde{b}\right)$ is a decreasing function. In the constrained problem the planner maximizes $\mathbb{W}$ subject to the constraint

$$x=\frac{\beta\left(1-\alpha-\widetilde{b}\right)}{\beta+(1-\beta)\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\widetilde{b}}\right]^{-1}}$$

Substituting this into the objective function and rearranging, we obtain

$$\mathbb{W}^c=\left[1-\beta+\frac{\alpha}{1-\alpha}\right]\left[\ln\left(1-\alpha-\widetilde{b}\right)-\ln\left(\beta\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\widetilde{b}}\right]+1-\beta\right)\right]+\beta\mathbb{E}_z\ln\left[\alpha z+\widetilde{b}\right]+\frac{\alpha}{1-\alpha}\ln\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\widetilde{b}}\right]+t.i.p.$$

where $t.i.p.$ denotes terms independent of $\widetilde{b}$. The first order necessary condition for an interior optimum is

$$\begin{aligned}\frac{d\mathbb{W}^c}{d\widetilde{b}} &= \left[1-\beta+\frac{\alpha}{1-\alpha}\right]\left[-\frac{1}{1-\alpha-\widetilde{b}}+\frac{\beta\mathbb{E}_z\left[\frac{\alpha z}{\left(\alpha z+\widetilde{b}\right)^2}\right]}{\beta\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\widetilde{b}}\right]+1-\beta}\right]+\beta\mathbb{E}_z\left[\frac{1}{\alpha z+\widetilde{b}}\right]-\frac{\alpha}{1-\alpha}\frac{\mathbb{E}_z\left[\frac{\alpha z}{\left(\alpha z+\widetilde{b}\right)^2}\right]}{\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\widetilde{b}}\right]}=0\\ &= \mathcal{D}\left(\widetilde{b}\right)-\frac{\alpha}{\beta(1-\alpha)}\left[\frac{1-\frac{\beta(1-\alpha)}{\alpha}\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\widetilde{b}}\right]}{\beta\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\widetilde{b}}\right]+1-\beta}\right](1-\beta)\beta\frac{\mathbb{E}_z\left[\frac{\alpha z}{\left(\alpha z+\widetilde{b}\right)^2}\right]}{\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\widetilde{b}}\right]}=0\end{aligned}$$

We know that $1-\frac{\beta(1-\alpha)}{\alpha}\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\widetilde{b}}\right]>0$, since $\frac{\alpha}{\beta(1-\alpha)}>1$ by Assumption 1 and $\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\widetilde{b}}\right]<1$. So $\mathcal{D}(\widetilde{b})=0$ at the unconstrained optimum, but $\mathcal{D}(\widetilde{b})>0$ at the constrained optimum. Since $\mathcal{D}$ is decreasing, it follows that $\widetilde{b}$ is higher at the unconstrained optimum than at the constrained optimum.

E.1 Relaxing the assumptions of log-utility and full depreciation

This section shows that the claims made in Proposition 1 generalize to a setting with homothetic time-separable utility functions and any depreciation rate $\delta\in[0,1]$. As is commonly known, for a single good, homothetic time-separable utility functions must take the form of CRRA utility functions, i.e. $u(c)=\frac{c^{1-\rho}}{1-\rho}$. Consider the young households first order condition for capital:

$$(1-\beta)\left[(1-\alpha)k^{\alpha}-k-b\right]^{-\rho}=\beta\mathbb{E}_z\left[\left(z\alpha k^{\alpha-1}+1-\delta\right)\left(zk^{\alpha}+(1-\delta)k+b\right)^{-\rho}\right]$$

Dividing by $k^{-\rho\alpha}$ and defining $\tilde{b} = b/k^{\alpha}$, we have

$$(1-\beta)\left[1-\alpha-k^{1-\alpha}-\tilde{b}\right]^{-\rho} = \beta\mathbb{E}_z\left[\left(z\alpha k^{\alpha-1}+1-\delta\right)\left(z\alpha+(1-\delta)k^{1-\alpha}+\tilde{b}\right)^{-\rho}\right]$$

Since the LHS is decreasing in k and $\tilde{b}$ while the RHS is increasing in both arguments, it is immediate that k is decreasing in $\tilde{b}$. Thus even with general CRRA preferences and less than full depreciation, increases in $\tilde{b}$ always crowd out capital, $dk/d\tilde{b} < 0$. Next we show that this decreases welfare to first order when $R = 1$. As in the main text, we assume that $R > 1$ in the absence of risk. To find conditions under which this is the case, we use the young households' first order condition for the riskless bond:

$$(1-\beta)\left[(1-\alpha)k^{\alpha}-k-b\right]^{-\rho} = \beta R\mathbb{E}_z\left[\left(zk^{\alpha}+(1-\delta)\,k+b\right)^{-\rho}\right]$$

Evaluating both Euler equations at zero risk and imposing $R > 1$, we have

$$\frac{\beta}{1-\beta} < \left[\frac{(1-\alpha)\,\delta-\alpha}{\alpha}\right]^{-\rho}$$

which is a generalization of Assumption 1 in the main text.[30] This assumption implies that if there is a sufficiently high level of risk in steady state, $R = 1$ in the absence of government debt. If this is in fact the case, then subtracting the two Euler equations we have $\mathbb{E}_z\left\{\left[\alpha z k^{\alpha-1}-\delta\right]\left[\alpha z k^{\alpha}+(1-\delta)\,k\right]^{-\rho}\right\} = 0$ which can be rewritten as:

$$\mathbb{E}_z\left[\alpha z k^{\alpha-1}-\delta\right]\mathbb{E}_z\left[\alpha z k^{\alpha}+(1-\delta)\,k\right]^{-\rho} + cov[\alpha z k^{\alpha-1}-\delta, \alpha z k^{\alpha}+(1-\delta) \quad = \quad 0$$

Since the covariance term is negative, it must be that $\alpha k^{\alpha-1}-\delta > 0$ in steady state: even when safe rates are zero, risky capital earns a positive expected return. Next, the welfare of the representative cohort is:

$$\mathbb{W} = \frac{1-\beta}{1-\rho}\left[(1-\alpha)\,k^{\alpha}-k-\tilde{b}k^{\alpha}\right]^{1-\rho} + \frac{\beta}{1-\rho}\mathbb{E}_z\left[z\alpha k^{\alpha}+(1-\delta)\,k+\tilde{b}k^{\alpha}\right]^{1-\rho}$$

Taking derivatives with respect to k, evaluating at $\tilde{b} = 0$ and using the capital Euler equation:

$$\left.\frac{d\mathbb{W}}{dk}\right|_{\tilde{b}=0} = (1-\alpha)\left\{(1-\beta)\left[(1-\alpha)\,k^{\alpha}-k\right]^{-\rho}\alpha k^{\alpha-1} - \beta\mathbb{E}_z\left[z\alpha k^{\alpha}+(1-\delta)\,k\right]^{-\rho} z\alpha k^{\alpha-1}\right\}$$

Using the Euler equation for bonds evaluated at $R = 1$, we can rewrite this as

$$\left.\frac{d\mathbb{W}}{dk}\right|_{\tilde{b}=0} = (1-\alpha)\,(1-\beta)\left[(1-\alpha)\,k^{\alpha}-k\right]^{-\rho}\left[\alpha k^{\alpha-1}-\delta\right] > 0$$

Thus the overall effect of a small increase in bonds on welfare is negative:

$$\begin{aligned}\left.\frac{d\mathbb{W}}{d\tilde{b}}\right|_{\tilde{b}=0} &= \left\{-(1-\beta)\left[(1-\alpha)\,k^{\alpha}-k-\tilde{b}k^{\alpha}\right]^{-\rho} + \beta\mathbb{E}_z\left[z\alpha k^{\alpha}+(1-\delta)\,k+\tilde{b}k^{\alpha}\right]^{-\rho}\right\}k^{\alpha} + \frac{d\mathbb{W}}{dk}\frac{dk}{d\tilde{b}} \\ &= 0 + \frac{d\mathbb{W}}{dk}\frac{dk}{d\tilde{b}} < 0\end{aligned}$$

[30] When $\rho = \delta = 1$, this reduces to Assumption 1.

where we again use the bond Euler equation with $R = 1$. Thus, even with more general preferences and technology than in our benchmark model with log utility and full depreciation, the main result in Proposition 1 goes through: it is not optimal to create government debt even when risk is high enough to make real interest rates slightly negative.

E.2 More general conditions under which safe assets crowd out capital and reduce welfare

The content of Proposition 1 holds even more generally. In this section, we prove Proposition 1 for general preferences and technology. We assume that a household maximizes some objective $E_z U(c_t^Y, c_t^O)$ subject to their budget constraints (1) and (2). U is a strictly concave function which is strictly increasing in both arguments. We assume that the production technology is neoclassical, represented by $f(k)$ where k denotes the capital labor ratio. Consequently, the real wage $w(k) = f(k) - f'(k)k$ is increasing in k and the average return on capital $\mathbb{E}_z R^k(z,k) = f'(k) + 1 - \delta$. Note that the average return of capital is decreasing in the quantity of capital: $\mathbb{E}_z \frac{\partial R^k(z,k)}{\partial k} = f''(k) < 0$. We also assume that $\frac{\partial R^k(z,k)}{\partial z} > 0$ and $\frac{\partial^2 R^k(z,k)}{\partial z \partial k} < 0$. These assumptions imply that the realized return on capital is increasing in realized productivity z and that a higher capital stock reduces the realized returns proportionally more for those those who draw high z. These assumptions are clearly satisfied in the case of purely multiplicative risk as in the paper. More generally, it holds more generally with less than full depreciation as in this case $R^k(z) = zf'(k) + 1 - \delta$.

Consider the problem of households who cannot trade the riskless bond; instead, they pay a transfer $b \geq 0$ when young and receive the same transfer when old. It is immediate that the allocations in this economy are the same as those in an economy where they can trade bonds and the government fixes the supply at b. For any preferences and technology, the date t solution to the young households choice of capital k_{t+1} can be expressed as some function:

$$k_{t+1} = \mathcal{S}(K_t, K_{t+1}, b)$$

where K_t denotes the aggregate capital at date t. Strict concavity of households preferences over consumption in both periods implies that $\partial \mathcal{S}/\partial b < 0$. Intuitively, higher b reduces marginal utility when old (and raises it when young), inducing households to invest less in capital holding aggregate variables constant. Similarly, provided that the production technology is such that higher capital stock increases wages, $\partial \mathcal{S}/\partial K_t > 0$, i.e, households respond to higher capital today by saving more. Given initial condition K_0, an equilibrium satisfies:

$$K_{t+1} = \mathcal{S}(K_t, K_{t+1}, b) \tag{43}$$

for all t and the steady state capital stock satisfies:

$$K^* = \mathcal{S}(K^*, K^*, b)$$

We call this steady state *stable* if there exists $\varepsilon > 0$ such that for all $K_0 \in (K^* - \varepsilon, K^* + \varepsilon)$, $K_t - K^*$ has the same sign as $K_0 - K^*$ and $\lim_{t\to\infty} K_t = K^*$, i.e. the economy returns monotonically to the original steady state K^* after a small perturbation.

Claim: In the neighborhood of any stable steady state, there is crowding out: $dK^*/db < 0$.

Proof: Notice that (43) implies that close to steady state, dynamics can be written as:

$$\frac{dK_{t+1}}{dK_t} = \frac{\mathcal{S}_1(K^*, K^*, b)}{1 - \mathcal{S}_2(K^*, K^*, b)}$$

where $\mathcal{S}_1$ and $\mathcal{S}_2$ denote the partial derivatives of the $\mathcal{S}$ function w.r.t the first and second arguments respectively. This steady state is stable if $\frac{dK_{t+1}}{dK_t} \in (0,1)$, i.e. $1 - \mathcal{S}_1(K^*, K^*, b) - \mathcal{S}_2(K^*, K^*, b) > 0$. Applying the implicit function theorem to the steady state, it is immediate that

$$\frac{dK^*}{db} = \frac{\mathcal{S}_b(K^*, K^*, b)}{1 - \mathcal{S}_1(K^*, K^*, b) - \mathcal{S}_2(K^*, K^*, b)} < 0$$

i.e., there is crowding out.

Next, we show that when risk is just high enough to make $R = 1$, increasing government debt strictly reduces the welfare of the representative cohort, which can be written as

$$\mathbb{W}(k, b) = \mathbb{E}_z \left[U \left(\omega(k) - k - b, R(k, z)k + b \right) \right]$$

The effect of an increase in debt is

$$\frac{d\mathbb{W}}{dk}\frac{dk}{db} = \underbrace{\mathbb{E}_z \left[-\frac{\partial U}{\partial c^Y} + \frac{\partial U}{\partial c^O} \right]}_{=0 \text{ since } R=1} + \frac{d\mathbb{W}}{dk} \underbrace{\frac{dk}{db}}_{<0}$$

Thus it suffices to show that the direct effect of a higher capital stock on welfare is positive. By the Envelope Theorem, the capital stock only affects welfare via factor prices:

$$\frac{d\mathbb{W}}{dk} = \mathbb{E}_z \left[\frac{\partial U}{\partial c^Y} \frac{\partial w}{\partial k} + \frac{\partial U}{\partial c^O} \frac{\partial R(z,k)}{\partial k} k \right]$$

Note that since the production function is neoclassical,

$$\frac{\partial w}{\partial k} = \frac{d}{dK}[f(k) - kf'(k)] = -f''(k)k = -\mathbb{E}_z \frac{\partial R(z,k)}{\partial k} k$$

Intuitively, since all income goes to capital or labor, an increase in the capital stock must raise labor income by exactly the same amount as it reduces average capital income. Thus, we have

$$\begin{aligned} \frac{d\mathbb{W}}{dk} &= -\mathbb{E}_z \left[\frac{\partial U}{\partial c^Y} \right] \mathbb{E}_z \left[\frac{\partial R(z,k)}{\partial k} k \right] + \mathbb{E}_z \left[\frac{\partial U}{\partial c^O} \frac{\partial R(z,k)}{\partial k} \right] \\ &> -\mathbb{E}_z \left[\frac{\partial U}{\partial c^O} \right] \mathbb{E}_z \left[\frac{\partial R(z,k)}{\partial k} k \right] + \mathbb{E}_z \left[\frac{\partial U}{\partial c^O} \right] \mathbb{E}_z \left[\frac{\partial R(z,k)}{\partial k} k \right] = 0 \end{aligned}$$

where we use the fact that $c^O(z)$ is increasing in z and $\frac{\partial U}{\partial c^O}$ is decreasing in $c^O(z)$, while $\frac{\partial R(z,k)}{\partial k}$ is decreasing in z, implying that the covariance between $\frac{\partial U}{\partial c^O}$ and $\frac{\partial R(z,k)}{\partial k}$ is positive. On average, higher capital stock decreases the income of old agents who own capital, and increase the income of young agents by the same amount. Since $R = 1$, the expected marginal utility of old and young agents is equal, so this redistribution from old to young would not change welfare if the loss in capital income was borne by all old agents equally.

However, since in fact capital income falls relatively more for those with a low marginal utility, the loss in expected utility of the old is smaller than the gain in utility of the young. In other words, a higher capital stock increases the share of safe income and reduces the share of risky income, increasing welfare in this incomplete markets economy.

F Constrained efficiency of zero debt

The ex-ante welfare of cohort t, given an allocation $\{k_t, b_t\}_{t=0}^{\infty}$, is

$$U_t = (1-\beta)\ln((1-\alpha)k_t^{\alpha} - k_{t+1} - b_t) + \beta\mathbb{E}_z \ln(\alpha z k_{t+1}^{\alpha} + b_{t+1}).$$

We consider a Ramsey planner who solves

$$\mathbb{U}(\phi) = \max_{\{k_{t+1}, b_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \phi_t U_t + \phi_{-1}\mathbb{E}_z \ln c_0^O(z) \tag{44}$$

subject to:

$$k_{t+1} = s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma)k_t^{\alpha} \quad , \quad \tilde{b}_t = \frac{b_t}{k_t^{\alpha}} \quad \text{and } k_0, b_0 \text{ given}$$

In the spirit of Negishi (1960), we call an allocation $\{k_t, b_t\}_{t=0}^{\infty}$ *constrained efficient* if it solves (44) for some sequence of Pareto weights $\{\phi_t\}$ with $\sum_{t=0}^{\infty} \phi_t < \infty$ with each $\phi_t \geq 0$ and at least one $\phi_t > 0$. The following Lemma characterizes conditions under which zero debt issuance is constrained efficient in this sense.

Lemma 5. *There exists $\sigma^{\diamond} > \underline{\sigma}$ such that, if $\sigma < \sigma^{\diamond}$ and $\widetilde{b}_0 = 0$, it is constrained efficient to choose $\tilde{b}_t = 0$ for all $t > 0$.*

Proof. Define $\sigma^{\diamond} = \sqrt{\ln\left[\frac{\alpha}{(\beta-\alpha)(1-\alpha)}\right]}$ and $k_{\max}^{1-\alpha} = s(0, 0, \sigma)$ We begin by showing that for all $\sigma \in [0, \sigma^{\diamond}]$, there exists at least one sequence of non-negative Pareto weights $\{\phi_i\}_{i=0}^{\infty}$ which satisfies absolute summability for which $k_t = k_{\max}$ and $\tilde{b}_t = 0$ for all $t \geq 0$ solve (44), while for $\sigma > \sigma^{\diamond}$, there is no sequence which of Pareto weights for which $(k_{\max}, 0)$ solves this problem.

Plugging the constraints into the objective function and rearranging yields:

$$\mathbb{U}(\phi) = \max_{\{\tilde{b}_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \phi_t \left\{(1-\beta)\ln\left[\left(1-\alpha-\tilde{b}_t\right) - s\left(\tilde{b}_t, \tilde{b}_{t+1}\right)\right] + \beta\mathbb{E}_z \ln\left[\alpha z + \tilde{b}_{t+1}\right]\right\}$$

$$+\phi_{-1}\beta\mathbb{E}_z \ln\left[\alpha z + \tilde{b}_0\right] + \sum_{t=0}^{\infty} \ln s\left(\tilde{b}_t, \tilde{b}_{t+1}\right)\left(\phi_t\beta + \sum_{j=t+1}^{\infty} \phi_j \alpha^{j-t}\right) + \text{constants independent of } \tilde{b}$$

The FOC is given by:

$$\phi_{t-1}\left\{(1-\beta)\frac{-s_2\left(\tilde{b}_{t-1},\tilde{b}_t\right)}{\left(1-\alpha-\tilde{b}_{t-1}\right)-s\left(\tilde{b}_{t-1},\tilde{b}_t\right)}+\beta\mathbb{E}_z\left[\frac{1}{\alpha z+\tilde{b}_t}\right]\right\}+\frac{s_2\left(\tilde{b}_{t-1},\tilde{b}_t\right)}{s\left(\tilde{b}_{t-1},\tilde{b}_t\right)}\left(\phi_{t-1}\beta+\sum_{j=t}^{\infty}\phi_j\alpha^{j-t+1}\right)$$
$$+\phi_t\left\{(1-\beta)\frac{-1-s_1\left(\tilde{b}_t,\tilde{b}_{t+1}\right)}{\left(1-\alpha-\tilde{b}_t\right)-s\left(\tilde{b}_t,\tilde{b}_{t+1}\right)}\right\}+\frac{s_1\left(\tilde{b}_t,\tilde{b}_{t+1}\right)}{s\left(\tilde{b}_t,\tilde{b}_{t+1}\right)}\left(\phi_t\beta+\sum_{j=t+1}^{\infty}\phi_j\alpha^{j-t}\right)\leq 0 \quad (45)$$

where

$$s_1\left(\tilde{b}_t,\tilde{b}_{t+1}\right)=\frac{\partial s\left(\tilde{b}_t,\tilde{b}_{t+1}\right)}{\partial\tilde{b}_t}=\frac{-\beta}{\beta+(1-\beta)\left(\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\tilde{b}_{t+1}}\right]\right)^{-1}}$$

and

$$s_2\left(\tilde{b}_t,\tilde{b}_{t+1}\right)=\frac{\partial s\left(\tilde{b}_t,\tilde{b}_{t+1}\right)}{\partial\tilde{b}_{t+1}}=-\frac{\beta\left(1-\alpha-\tilde{b}_t\right)}{\beta+(1-\beta)\left(\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\tilde{b}_{t+1}}\right]\right)^{-1}}\frac{(1-\beta)\left(\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\tilde{b}_{t+1}}\right]\right)^{-2}\mathbb{E}_z\left[\frac{\alpha z}{\left(\alpha z+\tilde{b}_{t+1}\right)^2}\right]}{\beta+(1-\beta)\left(\mathbb{E}_z\left[\frac{\alpha z}{\alpha z+\tilde{b}_{t+1}}\right]\right)^{-1}}$$

Evaluating (45) at $\tilde{b}_t=0$ for all $t\geq 0$ and rearranging yields:

$$\phi_{t-1}\frac{\beta(1-\alpha)e^{\sigma^2}}{1+(1-\alpha)(1-\beta)e^{\sigma^2}}\leq\alpha\sum_{s=0}^{\infty}\alpha^s\phi_{t+s} \quad (46)$$

Define $y_t=\sum_{s=0}^{\infty}\alpha^s\phi_{t+s}\in[0,\infty)$. So, $\phi_{t-1}=\alpha\left(\frac{1}{\alpha}y_{t-1}-y_t\right)$. Using these definitions, (46) can be written as:

$$\frac{\beta}{\alpha}\left[\frac{(1-\alpha)e^{\sigma^2}}{1+(1-\alpha)e^{\sigma^2}}\right]y_{t-1}\leq y_t$$

Since $y_t<\infty$ for any $\{\phi_s\}$ which satisfies absolute-summability[31], such a sequence $\{y_t\}$ exists iff

$$\frac{\beta}{\alpha}\left[\frac{(1-\alpha)e^{\sigma^2}}{1+(1-\alpha)e^{\sigma^2}}\right]<1$$

which holds as long as $\sigma<\sigma^\diamond$. Conversely, if $\sigma>\sigma^\diamond$, the above expression is strictly greater than one and no absolutely-summable positive sequence $\{\phi_t\}$ exists which satisfies (46). Finally, as is standard following Negishi (1960), an allocation is constrained efficient iff there exists Pareto weights $\{\phi_t\}$ such that the allocation solves the problem in (44) . So we are done. □

G Inefficiently low capital accumulation

Here we show that the allocation $(k_t,b_t)=(k_{\max},0)$ for all t, where $k_{\max}^{1-\alpha}=s(0,0,\sigma)$, features an inefficiently low level of capital, from the perspective of a social planner who can choose k_t and b_t without respecting individual savings decisions (but cannot redistribute within a generation). Starting from this allocation,

[31] y_t is the discounted sum of a absolutely-summable sequence and hence must be finite.

consider a deviation which increases k_t by ε and increases b_t by δ for every $t > 0$. We want to find (ε, δ) such that this deviation makes each cohort weakly better off. The effect of such a perturbation on the welfare of cohorts 0 and $t > 0$ can be written as:

$$\begin{aligned} d\mathbb{W}_0 &= -\left(\frac{1-\beta}{c^Y}\right)\varepsilon + \beta\mathbb{E}_z\left(\frac{\alpha^2 z k^{\alpha-1}\varepsilon + \delta}{\alpha z k^\alpha}\right) \\ d\mathbb{W}_t &= \left(\frac{1-\beta}{c^Y}\right)\left[-\delta - \varepsilon + \alpha(1-\alpha)k^{\alpha-1}\varepsilon\right] + \beta\mathbb{E}_z\left(\frac{\alpha^2 z k^{\alpha-1}\varepsilon + \delta}{\alpha z k^\alpha}\right) \end{aligned}$$

It is straightforward to show that $d\mathbb{W}_0 > 0$ if $\delta e^{\sigma^2} > \frac{\alpha}{\beta}\varepsilon$. Similarly, $d\mathbb{W}_t - d\mathbb{W}_0 > 0$ if $\frac{\alpha}{\beta}\varepsilon > \delta$, which implies that $d\mathbb{W}_t > 0$ if $d\mathbb{W}_0 > 0$. Thus, any sufficiently small (ε, δ) which satisfy:

$$\frac{\alpha}{\beta}\varepsilon = \left(\frac{1+e^{\sigma^2}}{2}\right)\delta$$

strictly increases welfare for all cohorts. Thus, the original allocation featured underaccumulation of capital. The deviation described here is not attainable in equilibrium for any debt policy: the original allocation already featured the highest possible level of capital attainable in equilibrium, namely $k_{\max}$.

H Proof of Proposition 2

The first claim follows from our analysis in section 4.1 which shows that when $R(\tilde{b}_L, \sigma_H) < 1/\Pi^\star$, then no full employment steady state can exist. To see why the second claim is true, first, we establish that the unemployment steady state is unique for γ sufficiently small. After that we construct an equilibrium in which $i_t = 0$ for all $t \geq 0$ and show that it is unique.

Steady states are characterized by:

$$\Pi^\star R\left(\tilde{b}_L l^{\alpha-1}, \sigma_H\right) = l^{-\frac{\alpha\gamma}{1-\gamma}}$$

When $\gamma = 0$, this equation has a unique solution. It follows immediately that for γ sufficiently close to zero, the steady state remains unique. Equilibrium with $i_t = 0$ for all $t \geq 0$ must satisfy the following conditions:

$$\begin{aligned} k_{t+1} + (1-\beta)\frac{\tilde{b}_L}{R_t}k_{t+1}^\alpha &= \beta\left[(1-\alpha)k_t^\alpha l_t^{1-\alpha} - \tilde{b}_L k_t^\alpha\right] && (47) \\ \Pi_{t+1}^{-1} = R_t &= \frac{1}{g\left(\tilde{b}_L l_{t+1}^{\alpha-1}, \sigma_H\right)}\alpha\left(\frac{k_{t+1}}{l_{t+1}}\right)^{\alpha-1} && (48) \\ l_t &= \min\left\{\left(\frac{k_t}{k_{t-1}}\right)^{1-\gamma} l_{t-1}^{1-\gamma}\left(\frac{\Pi_t}{\Pi^\star}\right)^{\frac{1-\gamma}{\alpha}}, 1\right\} && (49) \\ \left(\frac{\Pi_t}{\Pi^\star}\right) l_t^{(1-\alpha)\psi} &\leq 1 && (50) \end{aligned}$$

We proceed by assuming that (50) is satisfied with a strict inequality and that (49) holds with $l_t < 1$ for all t. Plug in (48) into (47):

$$\left[1+\left(\frac{1-\beta}{\alpha}\right)\tilde{b}_L l_{t+1}^{\alpha-1} g(\tilde{b}_L l_{t+1}^{\alpha-1},\sigma_H)\right]\frac{k_{t+1}}{k_t^\alpha}=\beta\left[(1-\alpha)l_t^{1-\alpha}-\tilde{b}_L\right]$$

Similarly, using (49) and (48):

$$\frac{k_{t+1}}{k_t^\alpha}=\alpha\frac{l_{t+1}^{\frac{1-\gamma(1-\alpha)}{1-\gamma}}}{l_t^\alpha}\frac{\Pi^\star}{g\left(\tilde{b}_L l_{t+1}^{\alpha-1},\sigma_H\right)} \tag{51}$$

Substitute the second equation into the first to get:

$$\begin{aligned}\Pi^\star\left[\frac{\alpha}{g\left(\tilde{b}_L l_{t+1}^{\alpha-1},\sigma_H\right)}+(1-\beta)\,\tilde{b}_L l_{t+1}^{\alpha-1}\right]l_{t+1}^{\frac{1-\gamma(1-\alpha)}{1-\gamma}} &= \beta\left[(1-\alpha)l_t-\tilde{b}_L l_t^\alpha\right] &(52)\\ LHS(l_{t+1}) &= RHS(l_t) &(53)\end{aligned}$$

It is easy to see that $LHS(l)$ is increasing and nonnegative, while $RHS(l)$ is negative for $l_t<l_{min}=\left(\frac{1-\alpha}{\tilde{b}_L}\right)^{\frac{1}{1-\alpha}}$, and positive and increasing after that. Furthermore, for γ sufficiently close to 0, the two curves have a unique intersection in $(0,1)$, as we now show. First let $\gamma=0$. Then after some algebra one can show that intersections of the two curves satisfy:

$$\frac{\Pi^\star}{\beta}\quad=\quad\mathbb{E}\left[\frac{1-\alpha+(\Pi^\star-1)\,\tilde{b}_L l^{\alpha-1}}{\alpha z+\tilde{b}_L l^{\alpha-1}}\right]$$

A sufficient condition for the derivative of the RHS with respect to l to be positive is that $\Pi^\star\leq\frac{1}{\alpha}$:

$$\begin{aligned}\frac{\partial}{\partial l^{\alpha-1}}\left\{\mathbb{E}\left[\frac{1-\alpha+(\Pi^\star-1)\,\tilde{b}_L l^{\alpha-1}}{\alpha z+\tilde{b}_L l^{\alpha-1}}\right]\right\} &= \mathbb{E}\left[\frac{\alpha z\,(\Pi^\star-1)-(1-\alpha)}{\left[\alpha z+\tilde{b}_L l^{\alpha-1}\right]^2}\right]\tilde{b}_L\\ &\leq (\alpha\Pi^\star-1)\,\mathbb{E}\left[\frac{1}{\left[\alpha z+\tilde{b}_L l^{\alpha-1}\right]^2}\right]\tilde{b}_L\end{aligned}$$

Thus, the solution for l is unique. Again, by continuity it follows that the solution is also unique for γ sufficiently close to 0.

It follows that at the unique intersection l^*, RHS cuts LHS from above, i.e. $RHS'(l^*)<LHS'(l^*)$. Thus if $l_0<l^*$, $LHS(l_1)=RHS(l_0)$ implies $l_1<l_0$, and so forth: $\{l_t\}$ is monotonically decreasing. The sequence cannot converge to any positive number: if it did converge, that limit would be another steady state, a contradiction. So eventually we must have $l_t<l_{min}$, which cannot be an equilibrium. By a similar argument, if $l_1>l_0$, we must eventually have $l_t>1$, which contradicts our assumption that the ZLB binds in every period. Thus the unique equilibrium with $i_t=0$ features $l_t=l^*$ in every period. It is straightforward to construct the rest of the equilibrium setting $l=l^*$. Iterating forwards on equation (51) delivers the dynamics of capital. Imposing $l_t=l^*$ in (51) for all $t\geq 0$ reveals that the path for capital is monotonically declining towards the new steady state. Plugging these into equation (48) we obtain a sequence of inflation rates. Finally since $l_t = l^* < 1$ for all $t \geq 0$, (50) is always satisfied for high enough ψ.

Why do we need to impose that ψ is large enough?. If the economy is hit with a large enough shock, the real return on bonds may actually be negative at date zero, as the economy's capital stock is far above its new steady state level. This in turn requires positive inflation in the short run, even though the economy will eventually arrive at a deflationary steady state. If ψ is too small, the monetary authority might be unwilling to keep rates at zero early on in the transition if the economy experiences positive inflation. In this case, no equilibrium exists, given our specification of fiscal and monetary policy. The economy desperately requires at least a few periods of negative real rates to smooth the transition to the new steady state, since capital is high in the short run, depressing interest rates even beyond the effect of the increase in risk. A monetary rule which will not accommodate temporarily negative real interest rates cannot even engineer a transition to a steady state with deflation and unemployment. Instead, employment spirals towards zero, eventually leaving the government unable to meet its fiscal obligations and such an equilibrium cannot exist. If ψ is sufficiently large, monetary policy is willing to tolerate short run inflation since output is below potential. In this case the equilibrium is described in the Proposition.

I Proof of Proposition 3

The problem of the steady state planner can be written as:

$$\mathbb{W} = \max_{k,\tilde{b},l,R,\Pi} (1-\beta)\ln\left[(1-\alpha)k^{\alpha}l^{1-\alpha} - \tilde{b}k^{\alpha} - k\right] + \beta\mathbb{E}_z \ln\left[\alpha z k^{\alpha} l^{1-\alpha} + \tilde{b}k^{\alpha}\right] \tag{54}$$

s.t.

$$\frac{k}{l} = k\left(\tilde{b}l^{\alpha-1}, \sigma\right) \tag{55}$$

$$R = R\left(\tilde{b}l^{\alpha-1}, \sigma\right) \tag{56}$$

$$\Pi = \Pi^{\star} l^{\frac{\alpha\gamma}{1-\gamma}} \tag{57}$$

$$\left(\frac{\Pi}{\Pi^{\star}}\right) l^{(1-\alpha)\psi} \leq 1 \tag{58}$$

$$R\Pi \geq 1 \tag{59}$$

$$(R\Pi - 1)\left[\left(\frac{\Pi}{\Pi^{\star}}\right) l^{(1-\alpha)\psi} - 1\right] = 0 \tag{60}$$

We begin by showing that the optimal choice always features full employment, $l = 1$. Take any putative solution $(k_*, l_*, \tilde{b}_*, R_*, \Pi_*)$ which features $l_* < 1$. Now consider a deviation in which $k' = \frac{k_*}{l_*}$, $l' = 1$ and $\tilde{b}' = \frac{\tilde{b}_*}{l_*^{1-\alpha}}$. Note that $(k', l', \tilde{b}')$ still satisfy (55)-(56) with the same $R\left(\tilde{b}', \sigma\right) = R\left(\tilde{b}_* l_*^{\alpha-1}, \sigma\right) = R_*$ and generate a higher level of inflation from (57). Since $l' = 1$, (58) is satisfied with equality and $\Pi' = \Pi^{\star}$. Since $\Pi' > \Pi_*$ and $R' = R_*$, (59) and (60) is satisfied. Thus, $(k', l', \tilde{b}', R', \Pi')$ is feasible if $(k_*, l_*, \tilde{b}_*, R_*, \Pi_*)$ is feasible. It is straightforward to check that this deviation increases social welfare by $-\ln l_* > 0$. Thus, in any solution to this problem we must have $l = 1$. As a result we can rewrite the problem as:

$$\mathbb{W} = \max_{k,\tilde{b}} (1-\beta) \ln \left[(1-\alpha) k^{\alpha} - \tilde{b} k^{\alpha} - k \right] + \beta \mathbb{E}_z \ln \left[\alpha z k^{\alpha} + \tilde{b} k^{\alpha} \right]$$

s.t.

$$\begin{aligned} k &= k\left(\tilde{b}, \sigma\right) \\ R\left(\tilde{b}, \sigma\right) \Pi^{\star} &\geq 1 \end{aligned}$$

This problem is identical to the problem in Proposition 1 except for the ZLB constraint which essentially puts a lower bound on $\tilde{b}$. This lower bound can be written as:

$$\tilde{b} > \tilde{b}_{\text{zlb}} \left(\sigma, \Pi^{\star}\right)$$

where $\tilde{b}_{\text{zlb}} \left(\sigma, \Pi^{\star}\right)$ is defined as the level of $\tilde{b}$ such that $R\left(\tilde{b}_{\text{zlb}} \left(\sigma, \Pi^{\star}\right), \sigma\right) \Pi^{\star} = 1$. Since the problem has a strictly concave objective, the result follows that the optimal $\tilde{b}$ satisfies:

$$\tilde{b} = \max\{\tilde{b}_{\text{zlb}}(\sigma, \Pi^{\star}), \tilde{b}_{\text{real}}(\sigma)\}$$

where $\tilde{b}_{\text{real}}(\sigma)$ denotes the optimal $\tilde{b}$ which solves the problem in Proposition 1 given σ. In particular if $\Pi^{\star} = 1$, then the level of $\tilde{b}$ required to ensure full employment is such that $R\left(\tilde{b}, \sigma\right) = 1$. From Proposition 1, we know that this level of $\tilde{b}$ is strictly higher than the optimal level absent nominal rigidities, i.e. $\tilde{b}_{\text{zlb}}(\sigma, \Pi^{\star}) > \tilde{b}_{\text{real}}(\sigma)$ whenever $\tilde{b}_{\text{zlb}}(\sigma, \Pi^{\star}) > 0$.

To prove the last part of the proposition, note that steady state welfare can be written

$$\mathbb{W}(k/l, \widetilde{b} l^{\alpha-1}) + \ln l$$

where $\mathbb{W}(k, \widetilde{b})$ denotes steady state welfare in the flexible price economy. Suppose that at the level of safe assets $\tilde{b}_*$ the ZLB binds, i.e. the corresponding steady state $(k_*, l_*, \tilde{b}_*, R_*, \Pi_*)$ features $l_* < 1$. Increasing $\widetilde{b}$ to $\widetilde{b}_{zlb}$ raises l from $l_* < 1$ to 1. When $\Pi^{\star} = 1$, it therefore reduces R from $l_*^{-(1-\alpha)\psi}$ to 1. Since $R(\widetilde{b} l^{\alpha}, \sigma)$ is increasing in its first argument, it follows that $\widetilde{b}_{zlb} < \widetilde{b} l_*^{\alpha} - 1$. We know that reducing $\widetilde{b} l^{\alpha-1}$ increases $\mathbb{W}(k/l, \widetilde{b} l^{\alpha-1})$ provided that $R \geq 1$. Since increasing $\widetilde{b}$ to $\widetilde{b}_{zlb}$ obviously increases $\ln l$, it follows that it increases steady state welfare.