On-the-job Search and the Productivity-Wage Gap *

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February 16, 2020

Abstract

We examine how worker and firm on-the-job search have differential impacts on the productivity-wage gap. While an increase in both worker and firm on-the-job search raise productivity, they have opposing effects on wages. Increased worker on-the-job search raises workers' outside options, allowing them to demand higher wages. Increased firm on-the-job search improves firms' bargaining position relative to workers' by raising job insecurity and the wedge between hiring and meeting rates. This allows firms to pass-through a smaller share of productivity to wages, enlarging the productivity-wage gap. Quantitatively, the model can account for the observed widening US productivity-wage gap over time.

Keywords: On-the-job search, Replacement hiring, Productivity-wage gap, Unemployment, Labor share
JEL Codes: E24, J63, J64

*The authors would like to thank Isabel Cairo, Keshav Dogra, Ross Doppelt, John Haltiwanger, Gueorgui Kambourov, Ryan Michaels and Laura Pilossoph for useful comments and suggestions. We also thank the seminar/conference participants at the Census Bureau, Carnegie Mellon University, EIEF, University of Edinburgh, FRB Cleveland, FRB Kansas City-CADRE Workshop, FRB St. Louis, MidWest Macro Pittsburgh, 9th European Sam Network Annual conference, Vienna Macro Workshop 2019. This paper was formerly titled "Replacement Hiring". The views expressed in this paper are entirely those of the authors. They do not necessarily represent the views of the Federal Reserve Bank of New York or the Federal Reserve System.
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1 Introduction

The last two decades have seen an increasing divergence between labor productivity and wages. Figure 1 shows that prior to 2000, real compensation per hour grew at roughly the same rate as real output per hour, i.e. labor productivity.[1] Post 2000, however, there has emerged a divergence between labor productivity and wage compensation. This gap between labor productivity and wage compensation - which we term the productivity-wage gap - continues to grow even as the unemployment rate continues to reach new lows in the aftermath of the Great Recession and the quits rate has surpassed its pre-recession level, suggesting that the increase in the gap is not due to slack labor markets. Given this backdrop, we ask instead how the incidence of firm on-the-job search and its impact on outside options can affect the productivity-wage gap.

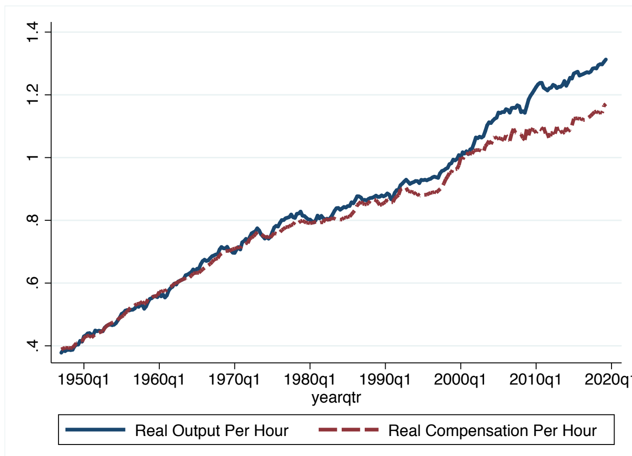


Figure 1: Labor productivity and real hourly compensation

Notes: (i) Data on Current Output Per Hour and Current Compensation per Hour comes from the U.S. Bureau of Labor Statistics Productivity and Costs database. We deflate both measures using the consumer price index (CPI). We normalize the deflated measures to be 1 in 2000Q1.

The way firm on-the-job search manifests itself is through replacement hiring - firms who seek higher quality applicants replace current workers with better workers. Importantly, over the same time period, there has been an upward trend in the fraction of total hires that are replacement hires.[2] Replacement hires are defined by the Census as hires that continue into the next period in excess of non-negative net employment change.[3] Using data from the Quarterly Workforce Indicators (QWI), Figure 2a shows that replacement hires as a fraction of total hires has increased from about 33% in the early 1990s to a high of about 41% in 2017.[4] While the time aggregation at a quarterly frequency implies that

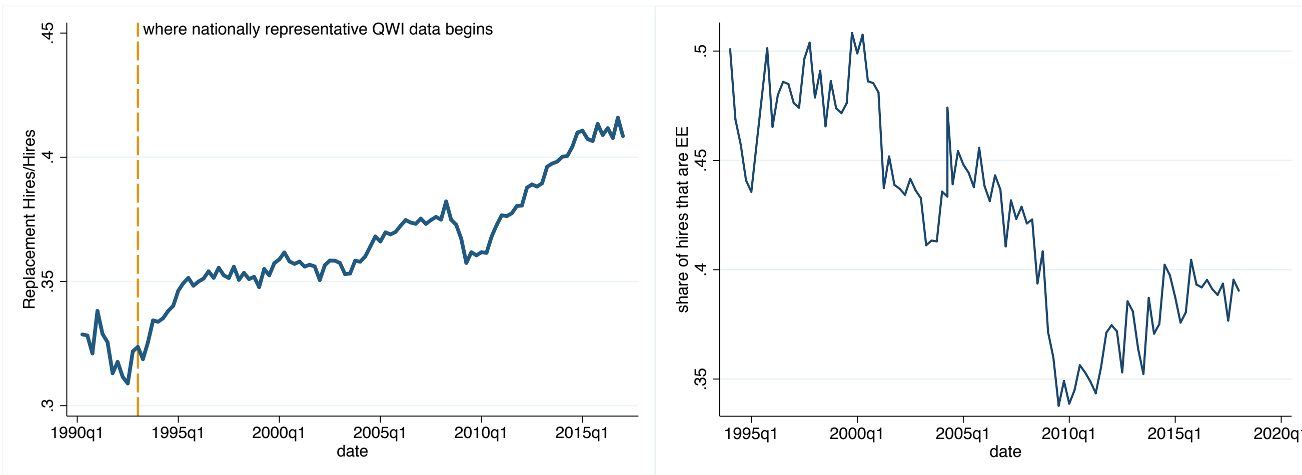
[1]To calculate real compensation per hour and real output per hour, we take current output per hour and current compensation per hour and deflate both measures using the CPI. Note that by using a common price index, the divergence in the productivity and wages stems not a difference in price deflators. Compensation includes direct payments to labor, e.g. wages, salaries, etc, and also includes wage supplements, e.g. private pension, health plans, etc.

[2]It is important to note that the increased share of replacement hires is not inconsistent with declining labor mobility and declining trends in job creation. In fact, as a fraction of average total employed, the replacement hiring *rate* has been declining over time. Figure 9a in Appendix A shows that the hiring rate has fallen faster than the replacement hiring rate. The sharper decline in total hires relative to replacement hires implies that replacement hiring is increasingly becoming a more important share of total hiring.

[3]See Section 2 for more details. All definitions are taken from `https://lehd.ces.census.gov/doc/QWI_101.pdf`.

[4]Information collected on replacement hiring as recorded in the QWI only begins from the 1990s.

replacement hires are also recorded whenever a firm re-fills a vacated position, Figure 2b shows that employment-to-employment transitions (EE hires) as a fraction of total hires has trended downwards. We further show in Section 2 that not all of replacement hiring in the data can be accounted for by quits. The rise in the replacement hiring share amid the decline in the ratio of EE hires to total hires suggests that firm on-the-job search may be an overlooked channel which is growing in importance.



(a) Replacement hires as a share of total hires

(b) EE hires/total hires

Figure 2: Trends in replacement hiring vs. worker job-to-job transitions

Notes: (i) The replacement hiring share is the ratio of replacement hires to hires. We formalize this definition in Section 2. (ii) The EE hires share is the number of employed individuals who moved to a new employer divided by hires. This measure is calculated using data from the Current Population Survey (CPS). To capture the numerator of this measure, we employ the same techniques as in Fallick and Fleischman (2004).

We view firm on-the-job search as a natural starting point for understanding how a wider productivity-wage gap could emerge. To this end, we build a model that features both worker and firm on-the-job search. In our model, search is random and firms pay a fixed cost to create a new vacancy. A vacancy in our model is synonymous with a job position being created. A vacancy or job position is long-lived and is not destroyed instantaneously. Rather, a vacancy continues to exist even if the firm fails to fill it immediately. Further, firms whose job positions have been filled can continue to meet applicants i.e, firms can conduct on-the-job search, so long as the job position has not been destroyed. When firms are allowed to search on-the-job, they seek applicants who are better matches than their current workers, and who can bring higher profits to the firm. In the same vein, workers in our model also conduct on-the-job search so as to meet vacancies who are better matches than their current firms. In their efforts to match with higher productivity applicants and firms, both firm and worker on-the-job search cause labor productivity to increase. Thus, an increased incidence of either worker or firm on-the-job search is associated with higher labor productivity.

What matters for the emergence of a productivity-wage gap is the *extent* to which productivity is passed-through to wages. If a smaller share of a one percent increase in productivity is passed through to wages, then wages increase by less with a productivity improvement and the productivity-wage gap is wider. Worker and firm on-the-job search have different implications for the pass-through of productivity to wages. A greater ease with which a worker can search on-the-job raises the worker's *effective outside option* - defined as the relative gain a worker observes from disagreeing to a match of particular productivity and continuing to search in unemployment. A higher effective worker outside option improves the worker's bargaining position relative to that of the firm and allows her to demand a higher share of productivity to be passed-through to wages.

In contrast, a greater ease with which firms can search on-the-job does the opposite. Using our theoretical framework, we uncover three channels through which firm on-the-job search depresses workers' effective outside options relative to that of firms and show how this in turn lowers the pass-through from productivity to wages. First, long-lived vacancies imply that firms have a positive option value from holding a vacancy. This positive option value raises a firm's effective outside option, allowing it to keep wages low when bargaining with the worker. Second, firms' ability to conduct on-the-job search raises workers' effective separation rate and hence the *job insecurity* they face. Increased job insecurity reduces employment spells and the worker's effective outside option, further allowing firms to pass-through a smaller share of productivity to wages. Finally, when firms can search on-the-job, the composition of vacancies comprises of both unfilled and filled vacancies. For an unemployed applicant to be hired at this latter type of vacancy, her productivity must be higher than that of the firm's incumbent worker. Thus, relative to meeting an unfilled vacancy, unemployed job-seekers must pass a higher bar before they are hired. This drives a larger wedge between hiring and meeting rates - lowering *measured matching efficiency* and in turn diminishing workers' effective outside options and their ability to capture a larger share of productivity. All three forces promote a wider productivity-wage gap.

To investigate the impact of firm vs. worker on-the-job on the productivity-wage gap, we conduct two separate exercises. First, we present some comparative statics. We analytically show that an increase in either the worker's or firm's ability to conduct on-the-job search raises average productivity, ceteris paribus. Intuitively, when both firms and workers have a higher ease of searching on-the-job, they can re-match more easily into higher productivity matches, causing average labor productivity to increase. The extent to which productivity is passed-through to wages, however, depends on the relative ease with which workers or firms can search on-the-job. We show that when firm on-the-job search is more prevalent, workers' effective outside options are reduced relative to the firm's option value because of the three aforementioned forces. This change in effective outside options allows firms to pass on a smaller share of productivity to wages, widening the productivity-wage gap. Our comparative static exercises highlight that a higher ease of worker on-the-job search serves to narrow the productivity-wage gap while a higher ease of firm on-the-job search does the opposite. The size of the productivity-wage gap thus depends on the relative prevalence of firm vs. worker on-the-job search.

Second, we calibrate the model to match labor market flows in the US as well as the replacement hiring share and EE hiring share across two time periods: pre- and post 2000. We use the year 2000 as our cut-off as the divergence in the productivity-wage gap became more severe post 2000. In conducting this exercise, we examine if our model, when calibrated to match the rise in the replacement hiring share, can at the same time replicate the observed wider productivity-wage gap. In our quantitative exercise, we measure replacement hiring as having occurred whenever (i) firms search on-the-job and (ii) whenever they refill a vacated position (due to exogenous separations into unemployment or quits from worker on-the-job search). Our model predicts that the higher replacement hiring share widened the productivity-wage gap by 12%, close to the observed increase of 8% in the productivity-wage gap over the time periods of interest.[5]. This wider gap arises due to the firm's higher option value of an unfilled vacancy post 2000, as well as due to a greater prevalence of firm on-the-job search relative to worker on-the-job search. We further use our calibrated model to identify the contribution of worker vs. firm on-the-job search towards the widening productivity wage gap. Our counterfactual exercises suggest that it is changes in the firms' ability to search on-the-job which is primarily responsible for the observed widening of the productivity-wage gap.

Related Literature Our paper contributes to the growing literature on the impact of replacement hiring. Two papers are closely related to ours. Both Mercan and Schoefer (2019) and Elsby et al. (2019) focus on the business cycle properties of vacancy posting and examine how replacement hiring and quits by workers necessitates a firm to re-fill a position. Importantly, the two aforementioned papers focus on worker on-the-job search while we examine the implications of on-the-job search by firms and workers on the productivity-wage gap. Separately, Menzio and Moen (2010) examine replacement hiring in the context that firms seek to insure workers against income fluctuations but cannot commit to not replacing current workers in a downturn with cheaper new hires. While their paper is concerned with characterizing the efficient wage contract, we examine instead, the implications firm on-the-job search has on wages in the absence of any aggregate productivity shocks. In related work, Kiyotaki and Lagos (2007) study a random search model which features both replacement hiring and worker on-the-job search. They examine the extent to which the decentralized economy can implement the planner's outcome when workers and firm both engage in Bertrand competition in terms of life-time utility offers. Importantly, their paper abstracts from wages while our paper is primarily concerned with how worker vs. firm on-the-job search can affect the productivity-wage gap. As such, our paper takes a stance on how wages are determined and in doing so, highlights how worker vs. firm on-the-job search have contrasting implications for pass-through and the gap between productivity and wages.

Although we study how worker vs. firm on-the-job search can affect the productivity-wage gap, our paper also has implications for the labor share. Intuitively, the divergence in labor productivity and compensation implies that a smaller share of total output accrues to labor. Karabarbounis and Neiman (2013) document that the labor share has declined across countries and argue that capital deepening is the primary factor behind this decline. Elsby et al. (2013) conduct a comprehensive study and find a strong negative relationship between import exposure and the labor share at the industry level. We add to this debate by highlighting a separate channel that can give rise to a lower labor share. When firms are more able to do on-the-job search relative to workers, the higher option value of firms relative to workers allows firms to pass-through a smaller share of productivity to wages. This results in a wider productivity-wage gap and consequently, a lower labor share.

Recent work by Autor et al. (2017) and Azar et al. (2017) suggest that product market concentration is associated with labor market concentration. When there are a few firms that dominate the product and labor market, firms internalize that they have market power and offer lower wages. Using a model of oligopsony, Berger et al. (2019) estimate the extent of firm market power and its implied impact on the labor share. Importantly, they find that local labor market concentration has actually declined over

[5] We measure the productivity-wage gap as the ratio of labor productivity to mean compensation. Since the our data for replacement hiring only begins in the 1990s, we only calculate the change in the productivity-wage gap between the periods 1990-2000 and post 2000.

the past 35 years, suggesting that the change in local labor market concentration would have implied an increase in the labor share and a greater pass-through of productivity to wages. Relative to these papers, we offer an alternative view of firm labor market power. Our notion of firm labor market power rests not on market concentration, but instead relates to the firm's option value and its ability to conduct on-the-job search. Our model suggests that so long as firms have a higher ease of on-the-job search relative to workers, the pass-through of productivity to wages is smaller. Our model is distinct from models of monosopny, where firms internalize how their hiring and wage setting decisions affect labor market outcomes. In our model, firms are small and take labor market conditions as given. Instead, it is the positive option value of firms and their ability to search on-the-job that endogenously leads to a weakening in workers' bargaining position and to a smaller pass-thorough of productivity to wages.

Our paper is also related to the recent literature on phantom vacancies. Cheron and Decreuse (2017) and Albrecht et al. (2017) argue that phantoms are vacancies that have already found a match and cannot generate any more new hires. The existence of phantoms lowers matching efficiency as unemployed job-seekers cannot convert a meeting with a phantom into a hire. We offer an alternative view: matched firms with unexpired vacancies can still generate hires. An unemployed job applicant who contacts a recruiting matched firm, however, must surpass the productivity of the incumbent worker before she is hired. As such, these long-lived vacancies which allow firms to conduct on-the-job search also lowers measured matching efficiency. Recent work using online vacancy job board data by Davis and de la Parra (2017) suggests that a non-trivial portion of job postings are "long-duration" postings which are continuously on the look-out for new applicants, giving support to our supposition that vacancies are long-lived and can re-match with multiple workers.

Finally, our paper is also related to the literature on long-lived vacancies. Fujita and Ramey (2007) and Haefke and Reiter (2017) consider models where job positions are long-lived and firms do not shut down immediately upon worker separation. Both of these papers demonstrate that the inclusion of long-lived vacancies in a labor search model can better replicate labor market flows in the data. Firms with unexpired job positions in these models only re-hire new workers when they are separated from their current worker. As such, these models do not address the issue of firm on-the-job search and its ramifications for the productivity-wage gap.

The rest of this paper is organized as follows. Section 2 discusses the data on replacement hiring. Section 3 introduces the model. Section 4 outlines our comparative static exercises and highlights how worker vs. firm on-the-job search affects wages and productivity. Section 5 presents our quantitative analysis while Section 6 contains a brief discussion about some assumptions of our model Finally, Section 7 concludes.

2 Data

Building on the underlying Longitudinal Employer Household Dynamics (LEHD) linked employee-employer database, the QWI provides information on key labor market outcomes. In particular, the QWI provides information at the state, industry and national level on the number of hires, separations, job gains and losses as well as average earnings. It should be noted that the QWI provides information at the *establishment level*. Thus, all measures mentioned below are the publicly available aggregated measures derived from the underlying micro-data on establishments. Hence, while we use the term 'firm' in this paper because we are interested in the consequences of firm on-the-job search on the productivity-wage gap, it should be recognized that in the data, the unit of observation is at the establishment level.

The QWI defines job gains or job creation at a firm as the non-negative change in employment within a quarter, which can be formally written as:

$$\text{Job Gains } = \max\left\{\text{Emp}_t^{\text{end}} - \text{Emp}_t^{\text{beginning}}, 0\right\}$$

In contrast, hires at a firm in quarter t is defined at the total number of new employees at a establishment that did not have earning in period $t-1$ but that reported earnings at that firm in period t. The measure of hires records the gross inflows into a firm, while the measure of job gains records the non-negative net employment change at the firm. Replacement hires are defined in the QWI as the hires that continue into the next quarter *in excess* of job gains at a firm. Using data from the QWI, we calculate the replacement hiring share as the fraction of hires that are replacement hires, i.e.

$$\text{Replacement Hiring Share} = \frac{\text{Replacement Hires}}{\text{Hires}} = \frac{\text{Hires At End - Job Gains}}{\text{Hires}}$$

where "Hires At End" refer to hires that continue into the next quarter, i.e. an individual who records earnings in periods t and $t+1$ but not in period $t-1$. Importantly, "Hires At End" are a sub-set of "Hires". The latter includes both hires that continue into the next quarter and individuals hired only for that particular quarter, i.e. the individual only has a record of earnings at time t. By definition, when there are zero hires that continue into the next quarter, there would be zero replacement hires recorded.[6] It is important to note that because replacement hires capture the hires that continue into next quarter in excess of job gains, the replacement hiring share is *not* equivalent to the ratio of separations to hires. This is because only a subset of separations are associated with replacement hires.[7] Thus while all replacement hires are associated with separations, not all separations are replacement hires.

Because a replacement hire always coincides with a separation, it is useful to distinguish between the different types of separations that can lead to a replacement hire. Firstly, a replacement hire can occur when the firm conducts on-the-job search and decides to hire a more productive applicant to replace its current incumbent worker. A replacement hire can also occur for reasons unrelated to firm on-the-job search. In particular, a replacement hire can occur whenever the firm re-fills a vacated position. Using the JOLTS micro-data, Elsby et al. (2019) focus on firms who have the same employment level τ periods

[6] This implies that the level of replacement hires is bounded below by zero. Thus, if a firm contracts and only observed separations, the QWI records zero rather than "negative" replacement hires at this firm.

[7] The following accounting identity from the QWI makes clear that replacement hires is not equal to total separations observed in the data:

$$\text{Hires } - \text{Separations} = \text{Job Gains } - \text{Job Losses}$$

Since replacement hires are only measured as the hires in excess of job gains which only counts non-negative net-employment change, replacement hires are not equal to total separations. To see this, consider the example of a firm who started the period with 1 worker. Suppose that worker left the firm and the firm hired a new worker to re-fill its vacated position. In addition, this new worker left the firm before the end of the period. In this example, the firm experienced a net employment change of -1, stemming from the 2 separations and 1 hire. Because that hire did not continue into the next period, the replacement hiring share at this particular firm would be equal to 0 since no hire continued into the next period. However, the ratio of separations to hires in this example would be equal to 2.

later and measure the cumulative hires rate (solid blue line) and cumulative quits rate (dashed-red line) at such firms. Importantly, since these firms observe zero net employment change, the cumulative hires is equal to cumulative separations and these cumulative hires represent replacement hires. While quits do affect the amount of replacement hiring, Figure 3 from Elsby et al. (2019) shows that not all replacement hires stem from refilling positions vacated by workers who quit. Rather, Figure 3 reveals that a non-trivial wedge exists between the cumulative hires rate and cumulative quits rates (plus other separations)[8], suggesting that a significant portion of replacement hiring also occurs alongside the event of a layoff.

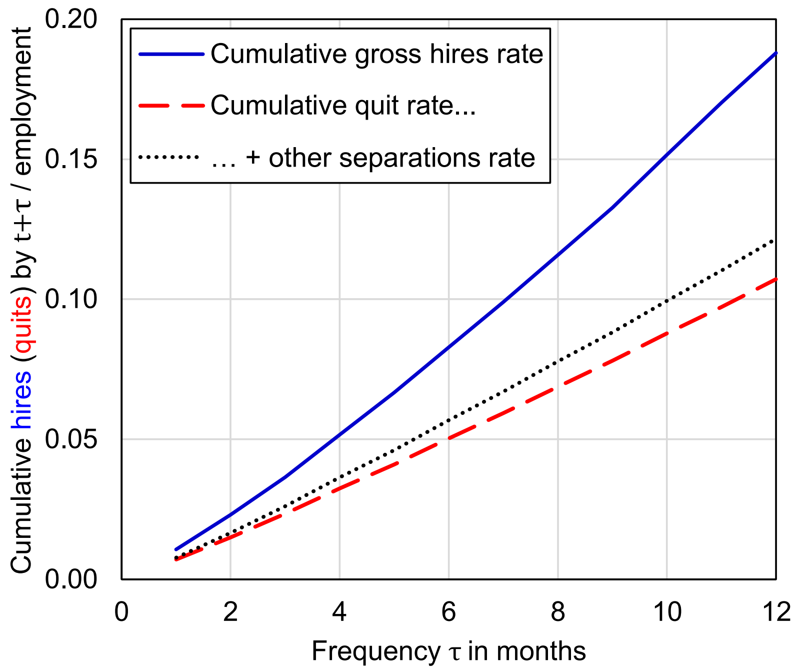


Figure 3: Both Layoffs And Quits Affect Replacement Hiring
Source: Elsby et al. (2019)

Our measure of replacement hires strictly follows that provided by the QWI. Elsby et al. (2019) use an alternative measure of replacement hires and define it as the minimum of gross hires and quits at a firm. This measure of replacement hires is not the same as the definition in the QWI but is consistent with the model presented in Elsby et al. (2019) which views replacement hires as being conducted whenever a worker quits. For our purposes, however, the definition of replacement hires as measured in the QWI is more appropriate since it captures replacement hires that are conducted both for the purposes of firm on-the-job search as well as for refilling a vacated position. To see this, consider a firm which had zero workers quit. The firm, however, decided to replace 1 worker with a higher productivity applicant. In this example, there is 1 hire, 1 fire and hence 1 replacement hire. Using a measure of replacement hires which is the minimum of gross hires and quits, however, would suggest that there are zero replacement hires when there are zero quits. As such, the QWI's definition of replacement hires

[8]The JOLTS data series defines other separations as separations stemming from retirements as well as discharges due to reasons of disability.

better suits our model's examination of replacement hiring that occurs for both firm on-the-job search reasons as well as for the purpose of refilling vacated positions. Having defined replacement hires, we use this definition to construct the replacement hiring share in Figure 2a.[9] Overall we find that the replacement hiring share[10] rose from about 0.33 in the 1990s to about 0.41 in 2017.[11]

Separately, Appendix A shows that the rise in the share of replacement hiring is not limited to a single industry's experience but occurs broadly across all industries. Further, Appendix A also shows that the results from a shift-share analysis suggest that the "within" component accounts for the bulk of the increase in the replacement hiring share. On average, older and larger firms tend to have higher replacement hiring shares. Nonetheless, cutting the data by firm age or firm size reveals that compositional changes towards older and larger firms can only explain 20% and 27% of the total change in the aggregate replacement hiring share respectively. The QWI also provides information by industry and by workers' education. Here, we find that the "between" component by industry or by worker education would have actually led to a negligible decline in the replacement hiring share. Instead, within each industry and education group, the replacement hiring share increased, accounting for most of the rise in the aggregated replacement hiring share. Finally, we also look at how recalls as a share of total hires have changed over time.[12] While some replacement hires can also be recall hires, Figure 12b in Appendix A makes clear that the replacement hiring share and the recall share of hires have differing trends, suggesting that recall hires are unlikely to be the driving force behind an increasing replacement hiring share. Finally, Figure 13 shows that that replacement hiring occurs both in high and low-paying industries (as measured by average earnings). One may have argued that if the high replacement hiring share was concentrated at industries with low average earnings, these may be jobs that workers frequently vacate in their search for better paying jobs, suggesting that worker on-the-job search as opposed to firm on-the-job search may be driving the rise in the replacement hiring share. We find that while the replacement hiring share is high in industries such as Retail Trade, and Accommodation and Food Services where average earnings is also low, the replacement hiring share is also high in high-paying industries like Management of Companies and Enterprises, and Finance and Insurance. The high replacement hiring share prevalent in high average earnings industries such as Management of

[9]It should be noted that Figure 2a records the trend in the replacement hiring share and not the replacement hiring rate where the denominator in the latter is total employment and the denominator in the former is hires. Further, the replacement hiring share should not be confused with churn in the economy which - following the definition in Burgess et al. (2000)- is defined as total worker flows (hires plus separations) in excess of job flows (hires minus separations). Churn rates at time t are then given by:

$$\text{Churn}_t = \frac{\text{Worker Flows}_t - \text{Job Flows}_t}{0.5(Emp_t + Emp_{t+1})} = \frac{\text{Hires}_t + \text{Separations}_t - (\text{Hires}_t - \text{Separations}_t)}{0.5(Emp_t + Emp_{t+1})} = \frac{2\text{Separations}_t}{0.5(Emp_t + Emp_{t+1})}$$

Since replacement hires are only a subset of all separations, replacement hiring can only be a portion of total churn.

[10]Importantly, Elsby et al. (2019), using their measure of replacement hires as the minimum of gross hires and quits, show in their paper that the replacement hiring share as captured by quits is relatively constant. This is suggestive that the growth in the replacement hiring share may not be driven by increased worker on-the-job search, and instead may be affected by firm on-the-job search.

[11]Our finding that the replacement hiring share rose over time is robust even if we use a different measure of the replacement hiring share. Specifically, if we only consider replacement hires as a fraction of all hires that continue into next quarter (as opposed to all hires that include individuals who were hired in a quarter but who did not stay on until the next quarter), this alternative replacement hiring share rose from 0.55 in the 1990s to about 0.60 in 2017.

[12]The QWI also provides information on recall hires, which are recorded whenever an individual i who did not have earnings at firm j in period $t-1$, but has earnings at firm j in period t as well as earnings at j in any previous period $t-2, t-3$ or $t-4$.

Companies and Enterprises[13] suggest that not all replacement hiring occurs for the purposes of refilling positions vacated by workers in low-paying industries with high turnover.

These findings indicate that the standard labor search model may be missing an important feature which is on-the-job search by firms. We outline in our model section how firm on-the-job search and worker-on-the-job search - both of which contribute to the incidence of replacement hiring - have contrasting implications for the emergence of a productivity-wage gap.

3 Model

Time is continuous and runs forever. The economy comprises of a unit mass of infinitely-lived workers who are ex-ante identical. All workers are risk neutral and discount the future at rate ρ. Workers can either be employed or unemployed. Unemployed workers receive flow utility b. The other agents in the economy are firms each of which can employ at most one worker at any date. A firm-worker pair with match quality x produces x units of output at each date. The match quality $x \in [0, \overline{x})$ is drawn from a time invariant distribution $\Pi(x)$ at the time the firm and worker meet and remains constant for the duration of the match.[14]

Vacancies and Firms Search is random. A firm that decides to enter the market must incur a fixed cost χ to post a vacancy. Posting a vacancy in our model is synonymous with creating a job position. We will thus use the term vacancies interchangably with job positions. A filled job position is equivalent to a matched firm-worker pair. All firms enter the labor market initially as unfilled vacancies. Importantly, unlike the standard DMP setup, in our model, unfilled vacancies do not expire instantly. This implies that a vacancy that goes unmatched today can still contact an applicant in the future as long as the vacancy/job position has not been destroyed. In addition, firms who were previously matched to a worker but whose worker separated from them at exogenous rate s, can still transition to become unfilled vacancies so long as their job position has not been destroyed. Thus, firms can replace workers who separated from them without posting a new vacancy.

Because vacancies are synonymous with job positions in our model, firms whose vacancies have been filled and whose job position has not been destroyed, referred to as *recruiting matched firms*, can still continue to meet and accept new applicants, i.e. firms can conduct on-the-job search. If the matched firm chooses to replace its current worker with the new job applicant, it releases its current worker into unemployment. In the case where the worker leaves the firm and the firm is unable to find a replacement, recruiting matched firms become unfilled vacancies. If a firm with an unfilled vacancy hires a worker, it becomes a recruiting matched firm.

Finally, a vacancy or position is destroyed at rate δ. When an unfilled vacancy is exogenously destroyed, the vacancy ceases to exist, while when a currently filled vacancy experiences the same shock, both the existing match and the vacancy cease to exist. This shock can be thought of as a firm

[13] Occupations included in the Management of Companies and Enterprises as listed by the BLS are that of accountants and auditors, accounting clerks, financial managers, first-line managers of office and administrative support workers, as well as general and operations managers. See https://www.bls.gov/iag/tgs/iag55.htm for reference.

[14] The support of x is allowed to be unbounded above, i.e., $\overline{x}$ can be ∞. In fact, in our calibrated model, we assume that x is described by a log-normal distribution and hence, the support is unbounded above.

no longer needing a worker for a particular position.

Labor Market Both unemployed and employed workers can contact vacancies. The rate at which unemployed workers meet vacancies (both currently unfilled and filled) is denoted by p while λ_w fraction of the currently employed workers can conduct on-the-job-search, and hence the effective rate with which employed workers can meet vacancies is given by $\lambda_w p \leq p$. Similarly, an unfilled vacancy meets job-seekers (both currently unemployed and employed) at a rate q. Unlike in the standard model, λ_f fraction of recruiting matched firms, i.e. firms with currently filled vacancies, can also search on-the-job and meet applicants at an effective rate of $\lambda_f q \leq q$. The contact rates p and q are determined by a *meeting* technology which takes as its inputs total vacancies and total applicants:

$$M = v^{1-\alpha}\ell^{\alpha}$$

where $\ell = u + \lambda_w(1-u)$ denotes the mass of job-seekers and $v = v^u + \lambda_f v^m$ is the measure of vacancies that can be contacted. Here, vacancies include all the unfilled vacancies, v^u, and the fraction of the currently matched firms who receive an opportunity to search, $\lambda_f v^m$. Similarly, ℓ includes all the unemployed workers, u, and the fraction of currently employed workers $1-u$ who get a chance to search on-the-job, $\lambda_w(1-u)$.[15]

Importantly, meeting rates are not equivalent to hiring rates. In order for a meeting to result in a hire, both the firm and the worker must agree to form a match. If an unfilled vacancy and an unemployed worker meet, they form a match whenever the match-specific productivity drawn is above a threshold $\widetilde{x}$, which is determined in equilibrium. However, if the unemployed individual meets a firm searching on-the-job, the new match quality x drawn must be at least as large as its incumbent worker's match quality. Thus, although the rate with which an unemployed worker meets a filled and unfilled vacancy is the same, the probability with which she will be hired is (weakly) lower at currently filled vacancies. Similarly, if an unfilled vacancy meets a currently employed worker, they form a new match only if they draw a new match productivity higher than that of the employed worker's old match. Finally, a meeting between a currently employed worker and a filled vacancy results in a new match only if the new match-productivity drawn exceeds that observed in both existing matches.

Workers can be both *exogenously* and *endogenously* separated from firms. The former occurs at rate s, while the latter occurs whenever a firm searching on-the-job replaces their current worker with a better applicant. Similarly, a filled vacancy can be both exogenously and endogenously separated from their employee. Endogenous separations occur when the worker finds a better match while searching on-the-job. Having described the environment, we next describe the firms' problems.

3.1 Firm's Problem

Recruiting matched firms The value of a filled vacancy (also referred to as a *recruiting matched firm*) with current match quality x can be written as:

$$\rho J(x) = x - w(x) + \delta\left[J^0 - J(x)\right] + \Big(s + p^*(x)\Big)\left[J^u - J(x)\right] + R(x) \tag{1}$$

[15] Of course, consistency requires that $1-u = v^m$.

The firm receives current profits $x - w(x)$ and can undergo four possible events in the future. First, the vacancy/job position is destroyed at a rate δ and ceases to exist. The firm can create a new unfilled vacancy and its associated change of value is $J^0 - J(x)$ where J^0 denotes the value of creating a new vacancy. Second, it undergoes an exogenous separation at rate s and becomes an unfilled vacancy with the associated change in value $J^u - J(x)$. Third, the firm's current worker successfully searches on-the job and quits the current firm to join another match. The firm transitions into an unfilled vacancy when this event occurs. Denote $p^*(x)$ denotes as the rate at which a worker with current match-productivity x successfully finds a better job while searching on-the-job:

$$p^*(x) = \lambda_w p \left\{ \left(\frac{v^u}{v}\right) [1 - \Pi(x)] + \left(\frac{\lambda_f v^m}{v}\right) \left[[1 - \Pi(x)] F(x) + \int_x^{\overline{x}} [1 - \Pi(\varepsilon)] \, dF(\varepsilon) \right] \right\} \quad (2)$$

The events that lead a worker to endogenously separate from the firm can be described as follows: first, a currently employed worker meets a vacancy at rate $\lambda_w p$. Conditional on a meeting, the worker meets an unfilled vacancy with probability v^u/v and with probability $1 - \Pi(x)$ forms a new match as it draws a match quality higher than her current x. With probability $1 - v^u/v = \lambda_f v^m/v$, the worker meets a filled vacancy with match quality ε; with probability $F(x)$ the filled vacancy's current match quality is lower than x and the new match is formed only if the pair draw a new match-quality larger than the worker's current match-quality x (which occurs with probability $1 - \Pi(x)$). Here, $F(\cdot)$ denotes the endogenous cumulative distribution function of existing matched firm-worker pairs across match quality with $F(\widetilde{x}) = 0$ and $F(\overline{x}) = 1$. The last term $\int_x^{\overline{x}} [1 - \Pi(\varepsilon)] \, dF(\varepsilon)$ represents the probability that a new match is formed when the employed worker with current match-quality x meets a filled vacancy with match quality $\varepsilon > x$ and they draw a new match-quality larger than ε.

Finally, the last term in eq.(1) - $R(x)$ - denotes the expected value of on-the-job search by a firm with current match quality x:

$$\begin{aligned} R(x) &= \lambda_f q \Big[\left(\frac{u}{\ell}\right) \int_x^{\overline{x}} [J(y) - J(x)] \, d\Pi(y) \\ &+ \left(\frac{\lambda_w v^m}{\ell}\right) \left\{ \int_x^{\overline{x}} [J(y) - J(x)] \, d\Pi(y) F(x) + \int_x^{\overline{x}} \int_\varepsilon^{\overline{x}} [J(y) - J(x)] \, d\Pi(y) \, dF(\varepsilon) \right\} \Big] \end{aligned} \quad (3)$$

A recruiting matched firm meets a new worker while searching on-the-job at the effective rate $\lambda_f q$. Conditional on the meeting, the first term is the change in value associated with the event that the firm meets an unemployed worker (with probability u/ℓ), draws a new match-quality $y > x$, forms the new match and enjoys a change of value $J(y) - J(x)$. The term on the second line reflects the expected change in value when the recruiting matched firm meets an employed applicant and a new match is formed. The first-term on the second line reflects the event when the firm with current match-quality x meets an employed applicant who has match quality $\varepsilon < x$ with her incumbent firm (this occurs with probability $F(x)$). In this case, the employed applicant with match-quality ε is always willing to form the new match if the firm with match quality $x > \varepsilon$ is willing to do so. Similarly, the second term on the second line refers to the event whenever the firm with match quality x meets an employed applicant with match-quality $\varepsilon \geq x$. In this case, it is the worker's decision to form a match which is binding.

Firms with unfilled vacancies The value of an unfilled vacancy can be written as:

$$\rho J^{u}=\delta\left[J^{0}-J^{u}\right]+q\left\{\left(\frac{u}{\ell}\right)\int_{\widetilde{x}}^{\overline{x}}\left[J\left(y\right)-J^{u}\right]d\Pi\left(y\right)+\left(\frac{\lambda_{w}v^{m}}{\ell}\right)\int_{\widetilde{x}}^{\overline{x}}\int_{\varepsilon}^{\overline{x}}\left[J\left(y\right)-J^{u}\right]d\Pi\left(y\right)dF\left(\varepsilon\right)\right\} \quad (4)$$

An unfilled vacancy observes zero flow profits. The vacancy is destroyed at rate δ (associated with the change of value $J^0 - J^u$). At rate q, the firm meets an applicant; with probability $\frac{u}{\ell}$, the applicant is unemployed. The unemployed worker and unfilled vacancy form a match as long as the match-quality they draw, y, is above the reservation match quality, $\widetilde{x}$. The firm's gain from such a match is given by $J(y) - J^u$. The reservation match-productivity $\widetilde{x}$ is defined as the lowest value of x for which the firm's gain is non-negative:

$$J(\widetilde{x}) - J^u = 0 \quad (5)$$

In the complementary case, i.e. with probability $1 - u/\ell = \lambda_w v^m/\ell$, the unfilled vacancy meets an employed worker with current match-quality ε. They form a new match only if they draw a new match-quality $y > \varepsilon$ in which case the firm's change in value is $J(y) - J^u$. Here, the composition of job-seekers affects the rate with which an unfilled vacancy becomes filled - holding all else fixed, a greater fraction of employed job-seekers searching on-the-job lowers the hiring rate of an unfilled vacancy since the employed worker only moves to a new match if the match quality drawn is higher than the existing value she shares with her incumbent firm.

Free Entry Under free-entry, the value of creating a new vacancy is driven down to 0:

$$J^0 = -\chi + J^u = 0 \quad (6)$$

which implies that the value of an unfilled vacancy $J^u = \chi > 0$, i.e., an unexpired vacancy provides the firm with a positive option value and affords the firm the ability to continue searching tomorrow even if it rejects or fails to meet a worker today. Further - and as we discuss subsequently - this positive option value raises the recruiting matched firm's effective outside option when bargaining with the worker, allowing it to bargain wages down.

3.2 Worker's Problem

Unemployed workers The value of a worker from unemployment, U, can be written as:

$$\rho U \;=\; b+p\left\{\left(\frac{v^{u}}{v}\right)\int_{\widetilde{x}}^{\overline{x}}\left[W\left(y\right)-U\right]d\Pi\left(y\right)+\left(\frac{\lambda_{f}v^{m}}{v}\right)\int_{\widetilde{x}}^{\overline{x}}\int_{\varepsilon}^{\overline{x}}\left[W\left(y\right)-U\right]d\Pi\left(y\right)dF\left(\varepsilon\right)\right\} \quad (7)$$

where $F(\varepsilon)$ denotes the fraction of recruiting matched firms whose worker possesses match quality ε or lower and $W(y)$ is the value of a worker who is employed at a recruiting firm with match quality y.

The value of unemployment can be decomposed into two terms: b, the flow utility associated with home production and the second term in equation (7) which denotes the expected change in value that the worker enjoys in the event that he transitions to employment in the future. At a rate p, an unemployed worker meets a vacancy. With probability v^u/v, this vacancy is currently unfilled and the worker is accepted whenever he draws a match quality higher than $\widetilde{x}$. However, with probability $\lambda_f v^m/v$, the unemployed worker encounters a recruiting matched firm and is only hired when she draws a match quality y that is higher than the incumbent's value. The second term inside the parenthesis captures the unemployed worker's change in value when she is accepted by a recruiting matched firm with current match quality ε weighted by the probability of meeting such a firm.

Unlike the standard model, the introduction of firm on-the-job search affects the composition of vacancies which in turn affects the worker's value of unemployment. Holding all else constant, a higher $\lambda_f v^m/v$ implies that unemployed workers are more likely to encounter recruiting matched firms as opposed to unfilled vacancies. This tends to lower the rate at which workers exit unemployment because recruiting matched firms require unemployed applicants to draw a match productivity above their incumbent worker's match quality. Thus, holding all else constant, a higher $\lambda_f v^m/v$ makes the the wedge between meeting and hiring rates larger, i.e. it lowers *measured matching efficiency*, lowering the worker's value of unemployment.

Employed workers The value of an employed worker at a recruiting firm with match quality x is:

$$\rho W(x) = w(x) - \Big(\delta + s + q^*(x)\Big)[W(x) - U] + H(x) \tag{8}$$

where $w(x)$ denotes the wages paid. There are three events that transition the worker into unemployment and hence result in a change in value of $U - W(x)$. First, at rate δ, the vacancy/position is destroyed, and the worker transitions to unemployment. Second, the worker is exogenously displaced into unemployment at rate s. Finally, the worker experiences a layoff whenever its firm meets a new applicant and forms a new match at rate $q^*(x)$ which is formally given by:

$$q^*(x) = \lambda_f q \left\{ \left(\frac{u}{\ell}\right)[1 - \Pi(x)] + \left(\frac{\lambda_w v^m}{\ell}\right)\left[[1 - \Pi(x)]F(x) + \int_x^{\overline{x}} [1 - \Pi(\varepsilon)]\, dF(\varepsilon)\right]\right\} \tag{9}$$

The events which add up to the worker being endogenously displaced into unemployment are as follows: the firm she is currently matched with meets an new applicant at rate $\lambda_f q$. Conditional on a meeting, the firm meets an unemployed applicant with probability u/ℓ and forms a match as long as the new match quality is higher than the current x. With probability $1 - u/\ell = \lambda_w v^m/\ell$ the firm meets a currently employed worker with match quality ε; with probability $F(x)$ the worker's current match quality, ε, is lower than x and a new match is formed if the pair draws a match quality, $y \geq x$. The last term $\int_x^{\overline{x}} [1 - \Pi(\epsilon)]\, dF(\epsilon)$ represents the probability that a new match is formed when the firm with match-quality x meets a currently employed worker with match quality $\varepsilon > x$, in which case the match is only formed if the new match-quality is larger than ε.

In addition, the value of an employed worker with match-quality x also includes the value of worker on-the-job search as denoted by $H(x)$:

$$H(x) = \lambda_w p \Big\{ \left(\frac{v^u}{v} \right) \int_x^{\overline{x}} [W(y) - W(x)]\, d\Pi(y) \\ + \left(\frac{\lambda_f v^m}{v} \right) \left[\int_x^{\overline{x}} [W(y) - W(x)]\, d\Pi(y)\, F(x) + \int_x^{\overline{x}} \int_\varepsilon^{\overline{x}} [W(y) - W(x)]\, d\Pi(y)\, dF(\varepsilon) \right] \Big\} \tag{10}$$

A worker searching on the job meets a vacancy at rate $\lambda_w p$. Conditional on meeting, the first term is the change in value experienced by the worker with current match-quality x when she meets an unfilled vacancy, draws a new match-quality $y > x$ and forms a new match. This results in a change of value of $W(y) - W(x)$. The term on the second line reflects the expected change in value when the currently employed worker with quality x meets a recruiting matched firm. The first-term on the second line reflects the event when the worker meets a currently filled vacancy who has a match quality $\varepsilon < x$. They form a match as long as the new match-quality is greater than x. Similarly, the second term on the second line refers to the event when the employed worker meets a filled vacancy with match-quality $\varepsilon \geq x$. In this case, a new match is only formed if the pair draws a match-quality $y \geq \varepsilon$.

Unlike the standard model, the introduction of firm on-the-job search introduces additional *job insecurity* for the worker through endogenous firm separations. Holding all else constant, if the ease of firm on-the-job search increases, i.e. λ_f rises, then $q^*(x)$ rises and workers' employment spells are shortened. This has the effect of lowering workers' employment values and feedbacks into a lower unemployment value, ρU, through the expected change in value when the unemployed form a match.

3.3 Surplus and Wage Formation

Wage Determination Wages are determined at each date via Nash Bargaining:

$$w(x) = \arg\max_{w(x)} \Big[J(x) - J^u \Big]^{1-\eta} \Big[W(x) - U \Big]^{\eta} \tag{11}$$

where $\eta \in [0, 1]$ denotes the bargaining power of a worker. Bargaining over wages takes place only after matches have been formed. This implies that whenever a recruiting matched firm chooses to hire a new applicant, he releases his current worker into unemployment prior to bargaining with the new applicant. Similarly, whenever a currently employed worker chooses to form a match with a different vacancy, she first vacates her current job. We further assume that there are no recalls. As such, when the firm with match quality x and a new applicant bargain over wages, the firm's outside option is simply the positive option value of an unfilled vacancy, J^u, and not $J(x)$. Similarly, for an employed individual with current match quality y, the worker's outside option is U and not $W(y)$.[16] As is well known, the

[16] Notice that this assumption is without loss of generality since wages are determined via Nash Bargaining each period without commitment. Even if firms bargained before separating with their current worker, and effectively used $J(y)$ as their outside option, it would mean that at the instant the next match is formed, it would revert to having J^u as its current outside option and would have to pay workers wages commensurate with equation (11). Of course, if the environment featured commitment by agents in the form of long-term contracts, then this would not be true and in that scenario, currently matched recruiting firms would offer a lower wage than unfilled vacancies for an applicant with the same match quality.

Nash bargaining solution implies that the surplus is split between firm and worker such that:

$$J(x) - J^u = (1-\eta)S(x) \qquad \text{and} \qquad W(x) - U = \eta S(x) \tag{12}$$

The surplus $S(x)$ of a match is the (discounted) total output produced by the firm-worker pair less their individual relative gains from continuing to search unmatched. Manipulating equations (1),(4),(7) and (8), the surplus for a matched firm-worker pair with match quality x can be written as:

$$\Big(\rho + s + \delta + q^*(x) + p^*(x)\Big)S(x) \quad = \quad x - \Big[\rho U - \widehat{H}(x)\Big] - \Big[(\rho+\delta)\,\chi - \widehat{R}(x)\Big] \tag{13}$$

where $\widehat{H}(x) = H(x) + p^*(x)\eta S(x)$ represents the expected value the worker gets from conducting on-the-job search less the value of unemployment and $\widehat{R}(x) = R(x) + q^*(x)(1-\eta)S(x)$ represents the expected value the firm gets from conducting on-the-job search less the value of an unfilled vacancy.[17] Equation (13) makes clear that the discounted surplus of the match is given by output x less the worker's *effective* outside option, $\rho U - \widehat{H}(x)$, and less the firm's *effective* outside option, $(\rho+\delta)\chi - \widehat{R}(x)$. Because both workers and firms can search on-the-job, their *relative* gain from walking away from a match of quality x and continuing to search as an unmatched agent is not just the value of unemployment or the value of an unfilled vacancy. Because the worker can search on-the-job, she can continue to meet vacancies at rate $\lambda_w p$ and will re-match with firms with quality $y > x$. Hence, the relative gain to a worker from walking away from a match of quality x is the opportunity to meet vacancies at a higher rate of p as well as the potential to form matches between $\widetilde{x}$ to x. Similarly, because the firm can also search on-the-job, its relative gain from walking away from a match of quality x is the opportunity to meet applicants at the higher rate of q as well as the potential to make matches between $\widetilde{x}$ to x. As such, the relative gain of continuing to search as an unmatched agent for the firm and the worker is characterized by their *effective* outside options. In Section 4, we outline how changes in effective outside options affect the pass-through of productivity to wages.

3.4 Labor Market Flows

Having described the relevant value functions, we proceed to describe labor market flows next.

Unemployed The steady state rate of unemployment u satisfies:

$$q\left(\frac{u}{\ell}\right)\left[1 - \Pi\left(\widetilde{x}\right)\right] v^u = \left(s+\delta\right) v^m + 2q\left(\frac{\lambda_f v^m \times \lambda_w v^m}{\ell}\right) \int_{\widetilde{x}}^{\overline{x}} \left[1 - \Pi\left(z\right)\right] F\left(z\right) dF\left(z\right) \tag{14}$$

The LHS represents the outflows from the pool of unemployed. At rate qu/ℓ, an unfilled vacancy meets an unemployed worker and hires her if they draw a match quality above $\widetilde{x}$. Notice that there is no net outflow when a currently filled vacancy hires an unemployed worker, as it also releases its current worker into unemployment. This feature of firm on-the-job search distinguishes it from worker on-the-job search. Note that when workers conduct on-the-job search, they leave their current firm,

[17] In other words, $\widehat{H}(x)$ is the expected value of employment from worker on-the-job search less the value of unemployment. Similarly, $\widehat{R}(x)$ is the expected value of a recruiting matched firm from firm on-the-job search less the value of an unfilled vacancy. See Appendix B for more detail.

causing an unfilled vacancy to open up and creating the start of a vacancy chain.[18] Here, if a firm conducts on-the-job search and hires a worker out of unemployment, it does not create a vacancy chain nor does it affect the unemployment pool on net, since hiring a worker out of unemployment requires it to displace its incumbent worker into unemployment.

The RHS denotes the flows into unemployment. The first term on the RHS, $(s+\delta)v^m$, is the fraction of all currently employed workers who experience a exogenous separation s or who observe their position/vacancy being destroyed, δ. The second term on the RHS refers to the flows into unemployment when a currently employed worker forms a match with a currently filled vacancy. In this case, the employed worker displaces the filled vacancy's incumbent worker into unemployment.

Unfilled vacancies Since vacancies are long-lived, the stock of unfilled and unexpired vacancies, v^u in steady state is implicitly defined by:

$$q\left(\frac{u}{\ell}\right)\left[1-\Pi\left(\widetilde{x}\right)\right]v^u+\delta v^u=v^{new}+sv^m+2q\left(\frac{\lambda_f v^m\times\lambda_w v^m}{\ell}\right)\int_{\widetilde{x}}^{\overline{x}}\left[1-\Pi\left(z\right)\right]F\left(z\right)dF\left(z\right) \tag{15}$$

The LHS of (15) represents the outflow from the pool of unfilled vacancies. The first term on the LHS $q\left(\frac{u}{\ell}\right)\left[1-\Pi\left(\widetilde{x}\right)\right]v^u$ is the number of unfilled vacancies which met an unemployed worker and formed a match. When an unfilled vacancy poaches a currently employed worker, there is no net-outflow from the pool of unfilled vacancies since the worker leaves the pre-existing match, transitioning that vacancy into an unfilled vacancy. The second term, δv^u, represents the unfilled vacancies which are destroyed.

The RHS of (15) represents the inflow into the pool of unfilled vacancies. The first component of inflows is v^{new}, the flow of newly created vacancies. Importantly, v^{new} is not counted as part of the vacancies available for matching today.[19] In the continuous time limit, $\theta=(v^u+\lambda_f v^m)/\ell\equiv v/\ell$. Thus, workers can only match with existing/old vacancies. New vacancies only add to the stock of unfilled vacancies in the future. The second term, sv^m, denotes all matched vacancies which experience an exogenous separation with their current worker. Finally, the third term on the RHS represents the inflows that occur when a currently matched vacancy and a currently employed worker form a new match, causing the old vacancy which employed the worker to become unfilled.

[18] Both Elsby et al. (2019) and Mercan and Schoefer (2019) explore how worker on-the-job search can give rise to vacancy chains, which are defined as the phenomenon where vacancies beget more vacancies.

[19] The total vacancies available for matching at time t are given by $v_t=(1-\delta\Delta)(v^u_{t-\Delta}+\lambda_f v^m_{t-\Delta})+v^{new}_t\Delta$ where Δ is the length of one period, $(1-\delta\Delta)(v^u_{t-\Delta}+\lambda_f v^m_{t-\Delta})$ is the stock of unexpired vacancies from the end of period $t-\Delta$. v^{new}_t is the number of new vacancies posted per unit time. Since each period is Δ units long, the total number of new vacancies posted in period t is $v^{new}_t\Delta$. Thus, market tightness can be written as:

$$\theta_t=\frac{(1-\delta\Delta)(v^u_{t-\Delta}+\lambda_f v^m_{t-\Delta})+v^{new}_t\Delta}{u_{t-\Delta}+\lambda_w v^m_{t-\Delta}}$$

In the continuous time limit, $\Delta\to 0$, the term $v^{new}\Delta$ becomes vanishingly small implying that current vacancies available for matching at period t consist only of existing or *old* vacancies $v^u_t+\lambda_f v^m_t$.

Endogenous distribution of match-productivity The steady state distribution of matched firm-worker pairs across match qualities $F(x)$ is implicitly given by:

$$
\begin{aligned}
q\left(\frac{u}{\ell}\right)\left[\Pi\left(x\right)-\Pi\left(\widetilde{x}\right)\right]v^{u} \quad = \quad & \left(s+\delta\right)F\left(x\right)v^{m}+q\left(\frac{u}{\ell}\right)F\left(x\right)\lambda_{f}v^{m}\left[1-\Pi\left(x\right)\right]+p\left(\frac{v^{u}}{v}\right)F\left(x\right)\lambda_{w}v^{m}\left[1-\Pi\left(x\right)\right] \\
& +2q\left(\frac{\lambda_{f}v^{m}\lambda_{w}v^{m}}{\ell}\right)\left\{\left[1-\Pi\left(x\right)\right]F\left(x\right)+\int_{x}^{\overline{x}}\left[1-\Pi\left(\varepsilon\right)\right]dF\left(\varepsilon\right)\right\}F\left(x\right) \\
& +q\left(\frac{\lambda_{f}v^{m}\lambda_{w}v^{m}}{\ell}\right)\left\{\int_{\widetilde{x}}^{x}\int_{z}^{x}\left[\Pi\left(x\right)-\Pi\left(\varepsilon\right)\right]dF\left(\varepsilon\right)dF\left(z\right)\right. \\
& \left.+\int_{\widetilde{x}}^{x}\left[\Pi\left(x\right)-\Pi\left(z\right)\right]F\left(z\right)dF\left(z\right)\right\} \qquad \text{for } x\in\left(\widetilde{x},\overline{x}\right) \qquad (16)
\end{aligned}
$$

with $F(\widetilde{x}) = 0$ and $F(\overline{x}) = 1$. The LHS of (16) represents the inflow into the set of matched-vacancies with match quality between $\widetilde{x}$ and x and is the number of unfilled vacancies which match with an unfilled worker and draw a match quality $y \in [\widetilde{x}, x]$.

The RHS of (16) denotes the outflows from the same set. The first term on the RHS denotes the number of matched-vacancies with match-quality less than x, i.e. $F(x)v^m$, who experience an exogenous separation or destruction of the vacancy. The second term represents the number of currently matched-vacancies who successfully matched with an unemployed worker and drew a new match-quality above x, thus reducing the number of matches with match-quality below x. The third term is the number of currently employed workers with match-quality less than x who successfully match with an unfilled vacancy and draw a new match-quality greater than x. The second line of (16) describes the case where an employed worker with $\epsilon < x$ and a filled vacancy with $z < x$ meet, and they draw $y > x$, then two firm-worker pairs leave the measure of matched pairs with match quality less than or equals to x. The third (and fourth) line of (16) describes the case where an employed worker with $\epsilon < x$ and a filled vacancy with $z < x$ meet, and they draw $\max\{z, \epsilon\} < y \leq x$, then one firm-worker pair leaves the distribution of firm-worker pairs with match quality less than or equals to x.

The distribution of matches by quality $F(x)$ is informative about the replacement hiring share. Holding all else constant, a distribution $F(x)$ which is skewed towards low values of x, indicates that there is substantial room for matched-firms to find a better match and thus to conduct replacement hiring. Similarly, employed workers also have substantial room to find a better match by searching on-the-job. When these workers find better matches and leave the firm, the firm with an unfilled vacancy must find a replacement, again encouraging replacement hiring. In contrast, a $F(x)$ with matches concentrated around higher values of x, both matched firms and employed workers find it harder to find better matches, reducing replacement hiring.

3.5 Closing the Model

The entire model so far has been summarized by the surplus equations and the labor market flows. However, all these relationships depend critically on the reservation match quality, $\widetilde{x}$, and the job-filling rate q which in turn is a function of labor market tightness θ. Lemma 1 summarizes the key equations which pin down the equilibrium $(\tilde{x}, \theta)$:

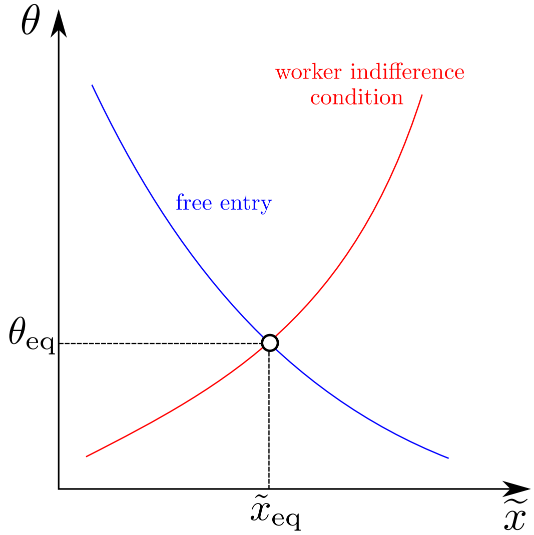


Figure 4: Equilibrium $\widetilde{x}$ and θ

Lemma 1. *In steady state, the equilibrium $\tilde{x}$ and θ are determined by the following equations:*

$$(\rho+\delta)\,\chi \;=\; (1-\eta)\,q\left[\left(\frac{u}{\ell}\right)\int_{\widetilde{x}}^{\overline{x}} S(y)\,d\Pi(y)+\left(\frac{\lambda_w v^m}{\ell}\right)\int_{\widetilde{x}}^{\overline{x}}\int_{\varepsilon}^{\overline{x}} S(y)\,d\Pi(y)\,dF(\varepsilon)\right] \tag{17}$$

$$\begin{aligned}\widetilde{x} \;=\;& \rho U-\eta\lambda_w p\left[\left(\frac{v^u}{v}\right)\int_{\widetilde{x}}^{\overline{x}} S(y)\,\pi(y)\,dy+\left(\frac{\lambda_f v^m}{v}\right)\int_{\widetilde{x}}^{\overline{x}} S(y)\,F(y)\,\pi(y)\,dy\right]\\ &+(1-\lambda_f)\,q\,(1-\eta)\left[\left(\frac{u}{\ell}\right)\int_{\widetilde{x}}^{\overline{x}} S(y)\,\pi(y)\,dy+\left(\frac{\lambda_w v^m}{\ell}\right)\int_{\widetilde{x}}^{\overline{x}} S(y)\,F(y)\,\pi(y)\,dy\right]\end{aligned} \tag{18}$$

where $S(x)$ and $F(x)$ are implicitly defined in equations (13) and (16). The asset value of unemployment ρU is given by:

$$\rho U = b+\eta p\left\{\left(\frac{v^u}{v}\right)\int_{\widetilde{x}}^{\overline{x}} S(y)\,d\Pi(y)+\frac{\lambda_f v^m}{v}\int_{\widetilde{x}}^{\overline{x}}\int_{\varepsilon}^{\overline{x}} S(y)\,d\Pi(y)\,dF(\varepsilon)\right\}$$

Proof. See Appendix B. □

Equation (17) is the free-entry condition where we have used $J^u = \chi$ from (6) and the solution to the Nash bargaining problem (12): $J(x)-\chi = (1-\eta)S(x)$ in (4). Equation (17) describes the minimum level of match quality for which an unfilled vacancy is willing to form a match for a given θ - or how *selective* a firm is as a function of labor market tightness. (17) implies a negative relationship between $\widetilde{x}$ and θ - firms are more selective when the rate of contacting applicants is high. In a slack labor market (low θ, high q), holding out for a better worker is relatively costless for the firm, and hence the firm raises its minimum level of match quality $\widetilde{x}$ for which it is willing to accept a worker. Conversely, in a tight labor market (low q), holding out for a better applicant is costly as the firm is unlikely to meet another applicant soon. As such, tight labor markets are associated with lower firm selectivity.

Equation (18) can be thought of as the *worker's indifference condition*: given a level of market tightness θ, (18) defines the reservation match quality for which a worker will be willing to exit unemployment and form a match.[20] The higher the value of unemployment, ρU, the more *selective* a worker is, i.e. a higher $\widetilde{x}$. Since a tighter labor market (higher θ) implies a higher value of unemployment ρU, (18) implies a positive relationship between $\widetilde{x}$ and θ. Figure 4 depicts the worker's indifference curve and shows the upward sloping relationship between θ and $\widetilde{x}$. Notably, both λ_w and λ_f affect the worker's indifference condition. A greater opportunity for the worker to search on-the-job (higher λ_w) makes workers less selective, because a worker can easily search on-the-job for a better match even if she accepts a low quality match out of unemployment. A higher λ_f, or greater ease with which a firm can search on-the-job also lowers the worker's reservation $\widetilde{x}$. A higher λ_f allows a firm to replace a worker easily, reducing the worker's value of remaining unmatched by pushing down the wage that a worker receives for any given match-quality x as we show next.

Figure 4 depicts the firm's free entry curve and the workers indifference condition in $(\theta, \widetilde{x})$ space. The equilibrium level of selectivity $\widetilde{x}$ and labor market tightness θ is given by the intersection of the two aforementioned curves describing workers' selectivity vs firms' selectivity respectively. Next, we describe how changes in key parameters affect equilibrium outcomes.

4 Forces at Play

Varying the ease with which firms and workers can search on-the-job (λ_f and λ_w respectively) has implications for the behavior of average wages and labor productivity. To identify exactly how a change in the ease of searching on-the-job by workers and firms affects average wages and labor productivity differently, it is useful to work with the two polar cases: where either only worker on-the-job search is operative ($\lambda_f = 0$) or firm on-the-job search is operative ($\lambda_w = 0$). It is useful to consider these special cases, because the model simplifies and admits many closed form expressions under these polar cases, allowing us to better understand the results from our quantitative exercise in Section 5 which works with the general model in which both $\lambda_w, \lambda_f \neq 0$

In what follows, we conduct comparative static exercises where we hold $\widetilde{x}$ and θ constant. We do this because changing either λ_w or λ_f causes both the free entry curve and the worker's indifference condition to shift and knowing what happens to equilibrium θ and $\widetilde{x}$ depends on the relative magnitude to which each curve moves. As such, the following comparative static results represent a *partial* equilibrium analysis. Of course, when we proceed to the quantitative analysis, we will also consider the general equilibrium feedback.

4.1 Worker On-the-Job Search Only

4.1.1 Productivity

We start by shutting off firm on-the-job search, i.e. $\lambda_f = 0$, and consider how variations in λ_w, the ease with which workers can search on-the-job, affect the productivity distribution as well as wages. In this special case, the distribution of match-quality among employed firm-worker pairs, (16), simplifies and admits a closed form:

$$F(x) = \frac{s+\delta}{(s+\delta+\lambda_w p\left[1-\Pi(x)\right])} \frac{\Pi(x)-\Pi(\widetilde{x})}{1-\Pi(\widetilde{x})} \tag{19}$$

[20](18) is derived by evaluating (13) at $x = \widetilde{x}$ and using $S(\widetilde{x}) = 0$.

A quick inspection of (19) reveals that for a given $(\widetilde{x}, \theta)$, $F(x \mid \lambda_w^{high})$ first-order stochastically dominates (FOSD) $F(x \mid \lambda_w^{low})$ for $\lambda^{high} < \lambda^{low}$. Since $F(x \mid \lambda_w^{high})$ FOSD $F(x \mid \lambda_w^{low})$, the average productivity is higher with a higher λ_w:

$$\int_{\widetilde{x}}^{\overline{x}} x dF(x \mid \lambda_w^{high}) dx \geq \int_{\widetilde{x}}^{\overline{x}} x dF(x \mid \lambda_w^{low}) dx$$

Intuitively, when workers have greater opportunity (higher λ_w) to conduct on-the-job search, they find it easier to move to higher match-quality jobs which pay higher wages. Consequently, more matches tend to be concentrated around higher x's. Overall, a higher λ_w allows workers to easily move to higher x jobs and this in turn raises average productivity.

4.1.2 Pass-through

Importantly, what matters for the productivity-wage gap is the extent to which productivity is passed through to wages. If a higher λ_w raised average productivity but kept the pass-through from productivity to wages constant, then there would be no change in the productivity-wage gap as average wages would rise together with average productivity. For a wider productivity-wage gap to emerge, the pass-through from productivity to wages must be lower. We measure pass-through as how much of a marginally higher x translates into higher wages, i.e., the rate of change in wages with respect to productivity $w'(x)$. To understand how the productivity-wage gap varies with λ_w, it is useful to examine what surplus, effective outside options, and pass-through look like in equilibrium in the limit case where $\lambda_f = 0$. The following Lemma describes the equilibrium outcomes for this case.

Lemma 2. *In the limit where $\lambda_f = 0$, the surplus of a match with quality x takes the form of:*

$$(\rho + \delta + s + \lambda_w p[1 - \Pi(x)]) S(x) = x - (\rho + \delta) J^u - \left[\rho U - \widehat{H}(x)\right] \tag{20}$$

Correspondingly, the worker's effective outside option in a match of quality x is given by

$$\rho U - \widehat{H}(x) = \widetilde{x} - (\rho + \delta) J^u + \lambda_w p \eta \int_{\widetilde{x}}^{x} S(y) d\Pi(y) \tag{21}$$

Finally, the extent to which improvements in x are passed through to wages are given by:

$$w'(x) = \eta + (1 - \eta) \lambda_w p \eta S(x) \pi(x) \tag{22}$$

Proof. See Appendix C.1. □

When only worker on-the-job search is operative, (20) makes clear that the (discounted) surplus of a firm-worker pair with match-quality x is given by the output x less the firm's outside option, $(\rho+\delta)J^u$, and less the worker's *effective* outside option, $\rho U - \widehat{H}(x)$. Because workers can search on-the-job, they can continue to meet firms at rate $\lambda_w p$ and will re-match with unfilled vacancies with whom they share match quality $y \in [x, \overline{x}]$. This implies that the worker's relative gain from disagreeing to a match of quality x and continuing to search in unemployment - in other words, her *effective* outside option - is affected by the foregone opportunity of matching with firms with quality from $\widetilde{x}$ to x. Equation (21) shows that the worker's effective outside option is equal to the reservation match quality, $\widetilde{x}$ less what must be given to the firm to ensure its participation, $(\rho+\delta)J^u$, and plus the amount the worker must be compensated for her foregone opportunity of matching with vacancies with quality ranging from $\widetilde{x}$ to x. Finally, (22) denotes the pass-through from productivity to wages.

Eq. (22) shows that for a given $\widetilde{x}$ and θ, a higher λ_w raises the pass-through from productivity to wages $w'(x)$.[21] To understand why pass-through is higher, it is useful to examine how the worker's effective outside option is changing with λ_w. First, observe that in agreeing to form a match of quality x, the compensation the worker must receive for her foregone opportunity of matching with firms with quality $\widetilde{x}$ to x, i.e. the last term on the RHS of (21), is rising in λ_w. In other words, a higher λ_w by increasing the employed worker's contact rate with vacancies, elevates the worker's bargaining position and raises the amount of compensation the firm must give to the worker to ensure her participation. At the same time, an increase in λ_w also weakens how much the worker must give the firm to ensure its participation. Eq. (4) shows that the firm's outside option, $(\rho+\delta)J^u$, is affected by the composition of job-seekers. When the composition of job-seekers tilts towards that of employed job-seekers- which is the case when λ_w is higher holding all else constant - the probability that an unfilled vacancy forms a match is lower. For an unfilled vacancy to successfully hire an employed worker, it must draw a match quality higher than that which the worker currently shares with her incumbent firm. A higher λ_w, holding all else constant, shifts the composition of job-seekers towards employed workers, reducing the average acceptance rate for an unfilled vacancy and thus lowering its value. The combined effect of increased compensation for the worker for her foregone opportunities as well as a diminished firm outside option implies that the worker's effective outside option is rising in λ_w, allowing the worker to extract a larger share of productivity to be passed into wages.

Overall, our comparative static exercise establishes that holding all else constant, a higher λ_w 1) raises average productivity but also 2) increases the pass-through of productivity to wages. This latter effect acts towards narrowing the productivity-wage gap.

4.2 Firm On-the-Job Search Only

4.2.1 Productivity

We now consider the polar case where only firms can search on-the-job, $\lambda_f > 0, \lambda_w = 0$. Again, this limit case permits us a closed form expression for the distribution of matched firm-worker pairs $F(x)$:

$$F(x) = \left(\frac{s+\delta}{s+\delta+\lambda_f q\left[1-\Pi(x)\right]}\right)\frac{\Pi(x)-\Pi(\widetilde{x})}{1-\Pi(\widetilde{x})} \tag{23}$$

As in the other polar case, (23) shows that when firms have a higher ease of searching on-the-job (higher λ_f), they can re-match more often and move into higher quality matches. Given $\widetilde{x}$ and θ, $F(x \mid \lambda_f^{high})$ first-order stochastically dominates $F(x \mid \lambda_f^{low})$ for $\lambda_f^{high} > \lambda_f^{low}$. As such, average productivity is higher when firms have a higher ease of conducting on-the-job search as they find it easier to meet new applicants and form higher quality matches.

[21] While $S(x)$ is also affected by λ_w, Appendix C.1 makes clear that $w'(x)$ is increasing in λ_w.

4.2.2 Pass-through

To understand how pass-through is affected by firm on-the-job search, we again list down the equilibrium outcomes when $\lambda_w = 0$.

Lemma 3. *In the limit where $\lambda_w = 0$, the surplus of a match of quality x is given by:*

$$(\rho + \delta + s + \lambda_f q[1 - \Pi(x)])\, S(x) = x - \rho U - \left[(\rho + \delta) J^u - \widehat{R}(x)\right] \tag{24}$$

the firm's effective outside option is given by:

$$(\rho + \delta) J^u - \widehat{R}(x) = \widetilde{x} - \rho U + \lambda_f q(1 - \eta) \int_{\widetilde{x}}^{x} S(y) d\Pi(y) \tag{25}$$

and pass-through of productivity to wages is given by:

$$w'(x) = \eta - \eta(1 - \eta)\lambda_f q S(x)\pi(x) \tag{26}$$

Proof. See Appendix C.2. □

Eq. (24) shows that the discounted surplus of a firm-worker pair with match-quality x is given by output of that match less the worker's outside option, ρU, and the firm's *effective* outside option $(\rho + \delta)J^u - \widehat{R}(x)$. When firms can conduct on-the-job search, they can continue to meet applicants at rate $\lambda_f q$ and will form new matches for any $y \in [x, \overline{x}]$. As such, the firm's relative gain or effective outside option if it chooses to walk away from a match of quality x and search as an unfilled vacancy is: $(\rho + \delta)J^u - \widehat{R}(x)$. (25) makes clear that the firm's effective outside option is given by the reservation match quality less what must given to the worker to ensure her participation and plus the amount that the firm must be compensated for foregoing the potential to match with workers of quality $\widetilde{x}$ to x. Finally, (26) captures the extent of pass-through from productivity to wages. As surplus is rising in x and wages are a function of surplus, wages are also increasing in x, implying that $w'(x) \geq 0$ for all x.[22]

As before, what matters for the productivity-wage gap is the pass-through from productivity to wages, $w'(x)$. Given $\widetilde{x}$ and θ, (26) shows that the pass-through $w'(x)$ is declining in λ_f. Intuitively, this arises because the firm's effective outside option is also increasing in λ_f. Consider firms who agree to a match of quality x. When firms have a greater ease of searching on-the-job, they require higher compensation for their foregone opportunity of matching with workers from $\widetilde{x}$ to x. The last term of equation 25 shows that the compensation a firm must receive for its foregone opportunity is rising in λ_f. At the same time, the outside option of workers, ρU, is declining when λ_f rises. Equation (7) which depicts the value of unemployment shows that the composition of vacancies affects this value. Given $\widetilde{x}$ and q, a higher λ_f increases the fraction of vacancies which are made up of firms searching on-the-job. Since unemployed workers are only hired by such vacancies when they draw a match quality

[22] One can differentiate equation 24 and show that the derivative of surplus with respect to x is positive:

$$\frac{dS(x)}{dx} = \frac{1}{\rho + s + \delta + \lambda_w p\,[1 - \Pi(x)]}\,[1 + (1 - \eta)\,\lambda_w p \pi(x)\, S(x)] > 0$$

Since wages increase with surplus, this implies that $w'(x) = \frac{dw(x)}{dS(x)} \frac{dS(x)}{dx} > 0$

higher than that of the firm's incumbent worker, the rate at which workers flow out of unemployment is lower. This lowers measured matching efficiency and reduces the value of unemployment. The reduced value of unemployment together with the fact that firms must be compensated more for their foregone opportunity causes the firm's effective outside option to be rising in λ_f. This improved bargaining position of firms stemming from their enlarged effective outside option allows them to extract more from surplus and hence results in a smaller pass-through from productivity to wages.

Our comparative static exercise shows that unlike the case of worker on-the-job search, a rise in the ease of firm on-the-job search widens the productivity-wage gap as it 1) shifts the distribution of matched firm-worker pairs to more productive matches raising average productivity, but 2) lowers the pass-through from productivity to wages. Because average wages rise by less when λ_f is high for a given $\widetilde{x}$ and θ, the productivity-wage gap is larger under a high ease of firm on-the-job search.

4.3 The Importance of Pass-through

More generally, (27) expresses the relationship between pass-through $w'(x)$, the relative ease of worker on-the-job search λ_w and the relative ease of firm on-the-job search λ_f:

$$w'\left(x\right) = \eta + \left(1-\eta\right)\eta\left[\lambda_w p\frac{v^u}{v} - \lambda_f q\frac{u}{\ell}\right] S\left(x\right)\pi\left(x\right) \tag{27}$$

Holding $\widetilde{x}$ and θ fixed, equation (27) shows that a higher λ_w tends to increase pass-through while a higher λ_f tends to decrease pass-through. Because in equilibrium, both $\widetilde{x}$ and θ would vary with changes in λ_f and λ_w, we now turn to solving out model numerically to identify the extent to which firm and worker on-the-job search can affect the productivity-wage gap.

5 Numerical Exercise

Our main goal is to examine if our model calibrated to match labor market flow rates can capture the increase in the productivity-wage gap. Since the divergence in the productivity-wage gap became more severe post 2000, we conduct two separate exercises and split the time periods into a pre-2000 period (1990-1999) and a post-2000 (2000-2017) period. We separately calibrate the parameters of our model to each time period's labor market flows.

5.1 Calibration

A period in our model is a month. Accordingly, the discount rate, ρ, is set to 0.004 to reflect an annual interest rate of about five percent. Although a period in our model is a month, we time-aggregate our model-generated moments to a quarterly frequency. We do so since the information about the replacement hiring share is measured at a quarterly frequency. The quarterly time aggregation allows us to capture replacement hires conducted for both the purposes of re-filling vacated positions as well as for firm on-the-job search reasons. Following the literature, we set the bargaining weight, η, to be 0.5.[23] We use information from the Business Employment Dynamics (BED) database and compute the

[23] See for example Gertler et al. (2016).

average monthly job destruction rate, δ for the period 1990-2017 to be 0.0235.[24] We hold constant the job destruction rate over the two time periods.[25]

The empirical literature has typically found that wages are log-normally distributed. Thus, we assume that the distribution of match quality, $\Pi(x)$ is given by the log of $N(-\sigma_x^2/2, \sigma_x^2)$. Overall, this leaves us with six key parameters to calibrate $\{\lambda_f, \lambda_w, b, s, \sigma_x, \alpha\}$. We treat the fixed entry cost of job creation χ as a residual to be solved for within the model. Further, we calibrate α, the elasticity of the meeting function with respect to unemployment to information on labor market flows from the first time period (pre 2000) and hold it constant at its calibrated value for the second time period.

While all key parameters are jointly calibrated, we use the following moments to identify the parameters: to pin down λ_f, we target the quarterly replacement hiring share in each period. To pin down λ_w, we target the quarterly employment-to-employment (EE) hiring share,[26] while we target an unemployment insurance ratio of 0.7 to pin down b, the value of home production. We use information on the quarterly rate at which employed individuals exit into unemployment (EU rate), and the unemployment rate to pin down s and σ_x. Finally, to pin down α, we use information on the quarterly job-finding rate (UE rate). Because there is a distinction between gross and net flows in our model, targeting the exit rate from employment, the job-finding rate and the unemployment rate does not automatically make any one of the moments a linear combination of the others. The exit rate from employment is a function of total gross separations which is composed of both exogenous separations s and endogenous separations stemming from firm on-the-job search. The job-finding rate out of unemployment is affected by the rate at which the unemployed worker meets and is hired by both unfilled and filled vacancies. In contrast, the unemployment rate is affected only by net flows and is hence unaffected by the rate at which a filled vacancy hires an unemployed worker as an employed worker is simulataneously displaced into unemployment in such an event, giving rise to no change in the unemployment pool on net.

Using data from the Current Population Survey (CPS) on employment, unemployment and short term unemployment, we find, for the period 1990m1-1999m12,[27] that the average monthly exit probability of an employed individual is about 0.032 while the average monthly job finding probability of an unemployed individual is given by 0.44.[28] In continuous time, this would imply that workers leave employment with at a quarterly rate of $-3 * \log(1 - 0.032)$ and find jobs at a rate of $-3 * \log(1 - 0.44)$. The average unemployment rate during this period is about 5.4%. For the pre-2000 period, we find that

[24]The job destruction rate from the BED is computed on a quarterly basis and is calculated as the sum of all jobs lost in either closing or contracting establishments divided by total employment. For the time period of interest, the average quarterly job-destruction probability is 0.068. To convert this number to a monthly rate, we calculate δ as

$$\delta = -1/3 \ln (1 - 0.068) = 0.0235$$

[25]Our results do not vary very much if we allow δ to vary over the two time periods, since the monthly job destruction rate did not decline by a large amount across the two time periods, the implied monthly δ would be 0.0257 in the pre-2000 period while the implied δ in the post 2000 period would be 0.0222.

[26]In calculating the quarterly EE hiring share, we include in the numerator all monthly EE hires as well as E-U-E hires that occur in a quarter.

[27]Our calibrated parameters would not change much if we expand the sample to be between 1951m1 to 1999m12 as the average unemployment rate during that period is about 5.5%, the exit probability is about 0.034 and the job finding probability is about 0.45. The targeted labor market moments for the period 1990m1-1999m12 is not substantially different from the period 1951m1 to 1999m12.

[28] We calculate the unemployment outflow and inflows rates by following the method proposed in Shimer (2012).

the quarterly EE hiring share is about 48%.[29] Using data from the QWI, we calculate that the average share of replacement hires for the period 1990Q1 to 1999Q4 is about 0.35.[30] For the period post-2000, we find, using CPS data, an average unemployment rate of 6.1%, an average monthly exit probability of 0.023, an average job-finding probability of 0.32 and an EE hiring share of about 0.41. From the QWI, we find that the replacement hiring share in the latter time period is about 0.38.

Table 1: Model Parameters

Fixed Parameters			
Parameter	Description		Value
ρ	discount rate		0.004
η	bargaining weight		0.5
δ	average job destruction rate from BED		0.0235
Calibrated Parameters (1990-1999)			
Parameter	Value	Quarterly Targets	Model Moment
λ_f	0.181	replacement hiring share of 0.35	0.34
λ_w	0.087	EE hiring share of 0.48	0.46
b	0.611	70% UI ratio	0.68
s	0.019	exit rate of 0.098	0.097
σ_x	0.022	unemployment rate of 0.054	0.054
α	0.315	UE rate of 1.74	1.77
χ	3.690	residual from free-entry eqn	
Calibrated Parameters (post-2000)			
Parameter	Value	Quarterly Targets	Model Moment
λ_f	0.076	replacement hiring share of 0.38	0.38
λ_w	0.019	EE hiring share of 0.41	0.41
b	0.562	70% UI ratio	0.70
s	1.8e-8	exit rate of 0.070	0.070
σ_x	0.024	unemployment rate of 0.061	0.061
χ	7.514	residual from free-entry eqn	

Table 1 summarizes our calibrated parameters. A few items are note-worthy from Table 1. Firstly, and in line with the observed long-run trend decline in the hiring rate, our model requires that χ increase across the two time periods so as to reduce entry and capture the more muted UE rates in the second time period. This is qualitatively in line with the decline in firm entry rates observed in the data. Our model predicts that the creation of new job positions would have declined by 11% across the two time periods.[31] This lower entry is consistent with the decline in firm entry over this period of

[29]We calculate the EE hiring share for the period 1994 onwards since that is when the CPS introduced a question that allows us to track whether the employed individual was in a same or different job. This number is consistent with information reported in Fallick and Fleischman (2004) who find that for the period 1994-2003, about 40% of all new hires observed an employer change, i.e. an EE transition.

[30]It is not inconsistent for the EE hiring share to be larger than the replacement hiring share. Note that not all EE transitions require a replacement hire to be conducted. A worker may quit its existing firm for a different job and the firm may not re-fill the position. In that case, an EE hire would be recorded but no replacement hire would be observed.

[31]Using the laws of motion for unfilled vacancies and the unemployed, we can define new jobs as $v^{new} = \delta\,(v^u + 1 - u)$

time. Establishment openings in the BED declined by about 26% across these two time periods while establishment entry rate as measured in the Business Dynamics Statistics (BDS) declined by 18%

Secondly, because replacement hiring and EE transitions as a percentage of total employment are also falling across the two time periods, our calibrated exercise also requires λ_f and λ_w to fall across time with the decline in hiring rates. Notably, the extent to which λ_w falls is much greater than λ_f. Post 2000, λ_w is about 22% its original value in the pre-2000 period while λ_f is about 42% its original value. This together with the rise in χ, which has the effect of reducing firm entry and raising q, suggests that relative to workers, firms were more able to conduct on-the-job search. Indeed, given these parameter values, we find that effective rate at which workers and firms could conduct on-the-job search in the pre-2000 period were given by $\lambda_w \times p = 0.16$ and $\lambda_f \times q = 0.13$ respectively. In contrast, in the post 2000 period, workers' and firms' effective rate at which they could conduct on-the-job search were given by $\lambda_w \times p = 0.02$ and $\lambda_f \times q = 0.07$ respectively, suggesting that while both parties observed a decline in their ability to conduct on-the-job search, firm on-the-job search became more prevalent relative to worker on-the-job search in the second time period. Finally, in our calibration, the exogenous separation rate falls by a large amount in the post 2000 period to match the observed lower employment-to-unemployment (EU) transition rates in the post 2000 period (Crump et al., 2019). Since the fall in λ_w and λ_f also gives rise to lower endogenous separation rates, our calibration implies the exogenous rate of separation must fall by a larger extent to match the overall lower EU rate in the post 2000 period.

The declines in χ, λ_f and λ_w have implications for net employment growth - defined in the QWI as the difference between firm job gains and firm job losses. While we do not explicitly target the net employment growth in our model, our model predicts that net employment growth would have declined by 66%,[32] a value somewhat below the actual observed decline in net employment growth in the data of 95%.[33] Nonetheless, our calibrated values of λ_f, λ_w and χ across the two time periods largely capture the slowdown in hiring rates and net employment growth. Further, recent work by Hyatt and Spletzer (2016) show that the decline in hiring and separation rates over time have been accompanied with a shift in the tenure distribution towards longer tenure jobs. In particular, they note that the median tenure or job duration has gone up by about one full year from 3.5 years in 2000 to 4.6 years in 2012-2014. Constructing tenure as 1 over the probability that a matched firm-worker pair separates[34], our model predicts that the average tenure would have lengthened by about 1 year.[35] Figure 5a shows that firm-worker pairs with higher match quality x have longer tenure durations as both firm and worker are less likely to find a new partner whose match quality is better than that of their incumbent. Since

and further define the rate at which new jobs are created as $v^{new}/(1-u)$.

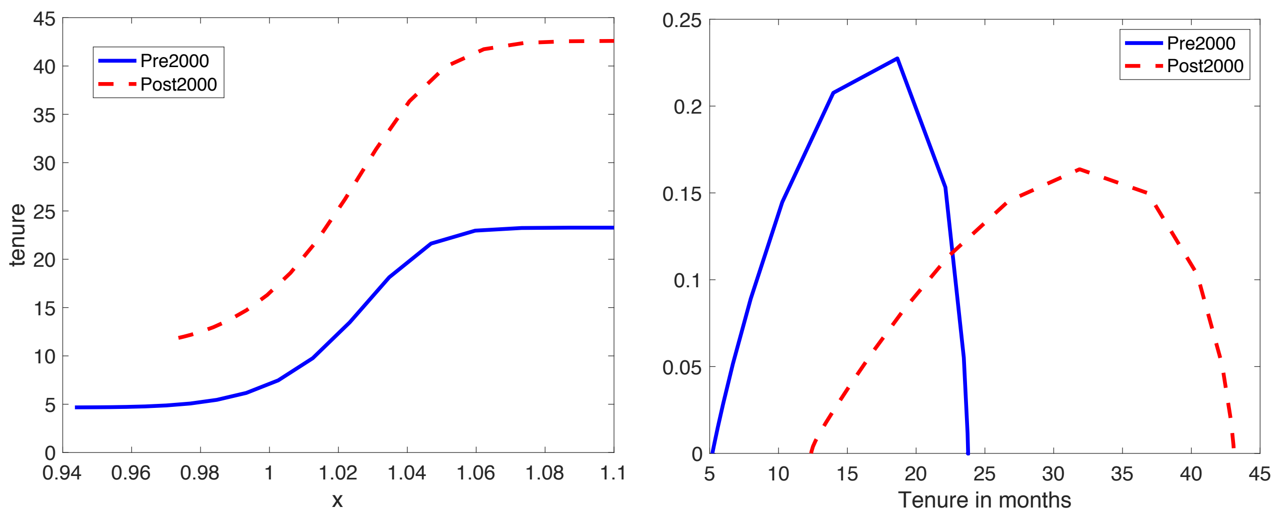
[32]In our model, job gains measure only hiring by unfilled vacancies since job gains is measured as $\mathbb{I}(\text{Hires} - \text{Replacement Hires}) > 0$. Job losses stem from both exogenous separations, s, and endogenous worker separations, $p^*(x)$, and from the job being destroyed, δ.

[33]Using the QWI data, net employment change is calculated as $\frac{\text{Job Gains} - \text{Job Loesses}}{\text{Employment}}$, where the measure of job gains and job losses is taken directly from the QWI.

[34]In our model, the average rate at which a firm-worker pair separates is given by $\text{job end rate} = \int_{\tilde{x}}^{\overline{x}} [p^*(x) + q^*(x)]\, dF(x) + s + \delta$. We convert this into a monthly probability by computing $\text{job end probability} = 1 - \exp(-\text{job end rate})$ and calculate the average tenure duration as 1 over the probability the job ends.

[35]We cannot match exactly the same median tenure observed in the data since the upper bound on tenures in our model is given by $1/\delta$ which works out to be about 43 months or 3.6 years. As aforementioned, δ is the exogenous job destruction rate, a number which we take directly from the BED data series. As such, for our purposes, it is more useful to compare if our model can replicate the change in average tenure duration.

wages are increasing in x, i.e. $w'(x) \geq 0$, this also implies that high wage jobs are also jobs with higher job security as workers are less likely to be endogenously displaced into unemployment when x is high. Figure 5b captures our model's predicted rightward shift in the tenure distribution, consistent with the fact that average tenures have increased over time across the two time periods. Hyatt and Spletzer (2016) also report that despite job tenures lengthening, there has been a decline in the returns to tenure. In Section 5.1.1, we show that our model can provide an explanation for this second fact of a decreased returns to tenure via a lower worker effective outside option.



(a) Tenure length by x

(b) Distribution of Matches Across Tenure

Figure 5: Model Predicted Tenure Distribution

Thus far, our model has been calibrated to capture labor market flows in and out of unemployment as well as the replacement hiring share and EE hiring share. We now ask whether model calibrated to match labor market flows can capture the divergence in the productivity-wage gap post 2000.

5.1.1 Model Implications for Productivity-Wage Gap

Having calibrated the model to match the observed changes in labor market flows, we now examine our model's implications for the productivity-wage gap. Table 2 shows our results. As aforementioned, all results in our model critically depend on equilibrium reservation match quality and labor market tightness, $(\widetilde{x}, \theta)$. The increase in χ and declines in λ_f and λ_w cause $\widetilde{x}$ to increase in the post 2000 period. The rise in χ and declines in λ_f and λ_w, however, have opposing effects on reservation match quality. An increase in χ has the impact of reducing the incentive to create jobs and encourages a leftward shift in the free entry curve as depicted by equation 17. When fewer jobs are created and labor market tightness is lower, workers respond by being less selective over reservation match quality. Thus, increase in χ work toward lowering $\widetilde{x}$.

The decreases in λ_f and λ_w have the opposite impact on the equilibrium reservation match quality. Intuitively, a lower λ_f and λ_w make it more difficult for the employed worker or the filled firm to re-match. Since it is difficult to move to higher quality matches in the future, firms and workers optimally raise their selectivity. In equilibrium, this latter effect dominates the impact of an increased

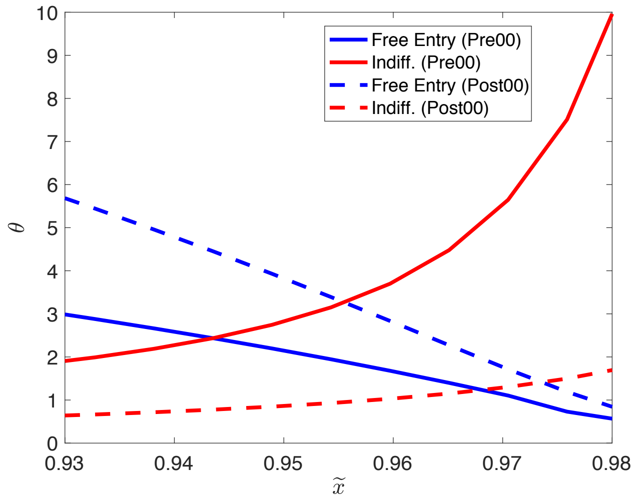


Figure 6: $\widetilde{x}$ and θ in Pre- and Post 2000 period

χ, causing both the free entry and worker indifference curves to shift rightward as depicted in Figure 6. In equilibrium, this causes $\widetilde{x}$ to increase in the post 2000 period. The higher reservation match-quality leads to a greater concentration of firm-worker pairs at higher levels of x. Overall, labor-productivity rises in the post 2000 period by about 1 percentage point. At the same time, Table 2 shows that the increase in χ and decline in λ_f and λ_w cause labor market tightness, θ, to fall by about 42%. This decline in θ raises reduces the worker's rate of meeting a vacancy and negatively affects the value of unemployment, ρU, causing it to fall by 11 percent in the post 2000 period.

Having described what happens in equilibrium to $\widetilde{x}$ and θ, we now come to our main question of interest: our model's predictions for the productivity-wage gap. We define the productivity-wage gap as the ratio of labor productivity, Y/N, to the average wage in the economy. In the data, the productivity-wage gap rose by about 8% between the two time periods. Our model predicts about a 12% increase in the productivity-wage gap across the two time periods, with one-twelfth of the increase stemming from a rise in labor productivity, Y/N, and the rest accounted for by a decline in average wages.

A few factors contribute to this wider productivity-wage gap. In particular, the increase in χ and the decline in ρU contribute towards elevating the firm's effective outside option while reducing that of the worker. In addition, (27) makes clear that when firms have a higher ease of conducting on-the-job search relative to workers, i.e. $\lambda_f > \lambda_w$, the pass-through from productivity into wages is smaller. In our model, the average pass-through declines by 21% in the post 2000 period. Together, the firm's enlarged effective outside option against a weaker worker outside option drive a smaller pass-through of productivity to wages, encouraging a wider productivity-wage gap.

We further examine the sources behind the decline in the worker's effective outside option. To do this, we look at two objects: 1) job insecurity, which we measure as the fraction of exits into unemployment that arise from endogenous separations, i.e.

$$\text{job insecurity } = \frac{\int_{\widetilde{x}}^{\overline{x}} q^*(x)dF(x)}{\int_{\widetilde{x}}^{\overline{x}} q^*(x)dF(x) + s + \delta}$$

Table 2: Non-targeted Model-Generated Moments

Pre vs. Post 2000				
	description	Pre-2000	Post 2000	Percent change
$\widetilde{x}$	reservation productivity	0.94	0.97	3.2
θ	labor market tightness	2.44	1.41	-42
$\lambda_f q$	firm OTJ contact rate	0.13	0.07	-49
$\lambda_w p$	worker OTJ contact rate	0.16	0.02	-85
job insecurity	fraction of EU that is endogenous	0.34	0.40	16
$\lambda_f v^m/v$	fraction of vacancies firm OTJ	0.50	0.64	28
matching efficiency	-	0.62	0.47	-23
ρU	worker outside option	0.88	0.79	-11
χ	firm outside option	3.59	7.53	110
$\int_{\widetilde{x}}^{\overline{x}} w'(x)dF(x)$	average pass-through	0.53	0.42	-21
Y/N	labor productivity	1.08	1.09	0.9
mean w	average wage	0.90	0.80	-11
$\frac{Y/N}{\text{mean}w}$	productivity-wage gap	1.19	1.35	12

and 2) measured matching efficiency which we measure as the average acceptance rate of the unemployed:

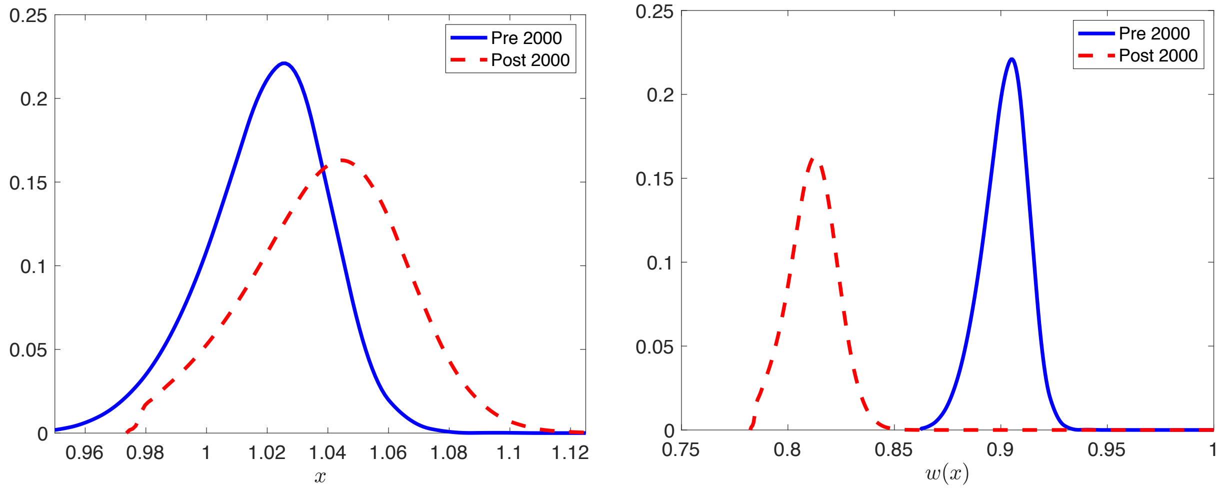
$$\text{measured matching efficiency} = \frac{v^u}{v}[1-\Pi(\widetilde{x})] + \frac{\lambda_f v^m}{v}\int_{\widetilde{x}}^{\overline{x}}[1-\Pi(z)]dF(z)$$

From the above equation, one can observe that measured matching efficiency is affected by the share of vacancies that constitute firms conducting on-the-job search, $\lambda_f v^m/v$.

In our results, the fraction of exits into unemployment which are endogenous rises by 16% as depicted in Table 2. This is because exogenous separation rates fall more relative to the firm's average replacement rate, $\int_{\widetilde{x}}^{\overline{x}} q^*(x)dF(x)$. This higher job-insecurity in turn reduces the value of unemployment and as such the worker's effective outside option. Next, looking at measured matching efficiency table 2 shows that the share of vacancies which are firms conducting on-the-job search $\lambda_f v^m/v$ rises by 28% in the post 2000 period.[36] This in turn causes a 23% decline in measured matching efficiency as unemployed applicants are only hired if they are more productive than the firm's incumbent worker. Declines in measured matching efficiency further worsen the worker's unemployment value and thus its effective outside option as workers find it harder to exit unemployment.

To illustrate how the changing effective outside options and smaller pass-through can differentially impact the distribution of realized productivity and wages, Figures 7a and 7b show how the productivity and wage distributions over the two time periods evolve. Focusing on the distribution of matched firm-worker pairs, Figure 7a shows that the realized productivity distribution in the post 2000 period has greater mass at higher productivities. Because declines in both λ_f and λ_w cause both firms and workers to be more selective, i.e. higher $\widetilde{x}$, this leads to more productive matches being formed on average,

[36]Even though λ_f declines in the post 2000 period, in the new equilibrium $\lambda_f v^m/v$. Intuitively, the greater fall in λ_w also lowers the rate at which at which filled vacancies lose workers (since $p^*(x)$ is lower) causing the fraction of filled vacancies that are searching on-the-job to increase.



(a) Productivity Distribution

(b) Wage Distribution

Figure 7: Model-implied Productivity and Wage Distribution

and labor productivity is higher. Despite more mass at higher productivities in the second time period, Figure 7b shows that wages were further skewed to the left in the post 2000 period. Depressed workers' effective outside options and lower pass-through of productivity to wages stemming fromfirms' higher ease of on-the-job search relative to workers contribute to the decline in average wages.

Importantly, in our calibration exercise we did *not* target any wage moments in the data. Using CPS data, we compute the portion of wages not explained by observables and analyze how the skewness in residualized log wages changed over time.[37] We find that in the data, the residual log wage distribution became less negatively skewed, rising from -0.096 in the pre-2000 period to -0.002 in the post 2000 period. The reduction in negative skewness suggests a larger concentration at lower wages in the post 2000 period and is qualitatively consistent with the leftward shift in the wage distribution we observe here. Taking 10000 random draws from our model's predicted wage distributions and applying the natural log, we compute that the skewness in our model's predicted log wage distribution pre-2000 is about -0.3. This skewness turns positive post 2000 to a value of 0.11.[38] One might argue that the leftward shift in the wage distribution is seemingly counterfactual, since real wages are growing in the data albeit slowly. The correct way to interpret this shift down is relative to trend growth. If we had assumed a balanced growth path where the exogenous match quality distribution, $\Pi(x)$, were allowed to shift rightward over time, our model would also predict wages growing over time as opposed to declining. However, our model would stress that wage growth would be slower precisely when firms have a higher effective outside option - either higher χ or higher λ_f relative to λ_w - as they can afford to pass-through a smaller portion of productivity to wages. Thus, the key take-away of our model is the prediction that

[37] Here, we run a mincer wage regression of log wages against education dummies, age and age-squared, a female dummy and race dummies, and compute the skewness of the residual wage distribution. We do this because our model has nothing to say about age gender, race and educational attainment, all of which are demographic factors that affect wages and that could changed over the two time periods.

[38] Clearly because we did not target wage moments, and because the residualized log wage from our regression has a mean of 0, the skewness values may not be the same between our model and the data. However, we emphasize that both our model and the data shares the feature that the wage distribution observed larger mass at lower wages across the two time periods.

the forces underpinning the extent of pass-through in turn affect the size of the productivity-wage gap.

Having discussed our model's predictions for the productivity-wage gap, we turn now towards identifying the role of λ_w and λ_f in affecting the rise in the productivity-wage gap.

5.2 The role of worker vs. firm on-the-job search

A key question is how much of the rise in the productivity-wage gap can be attributed to the change in λ_w vs. the change in λ_f. To answer this question, we now conduct the following counterfactual exercises. We keep all other parameters fixed at their post 2000 values and ask how much larger the productivity-wage gap would be if either λ_w or λ_f were separately held at their pre-2000 values.[39]

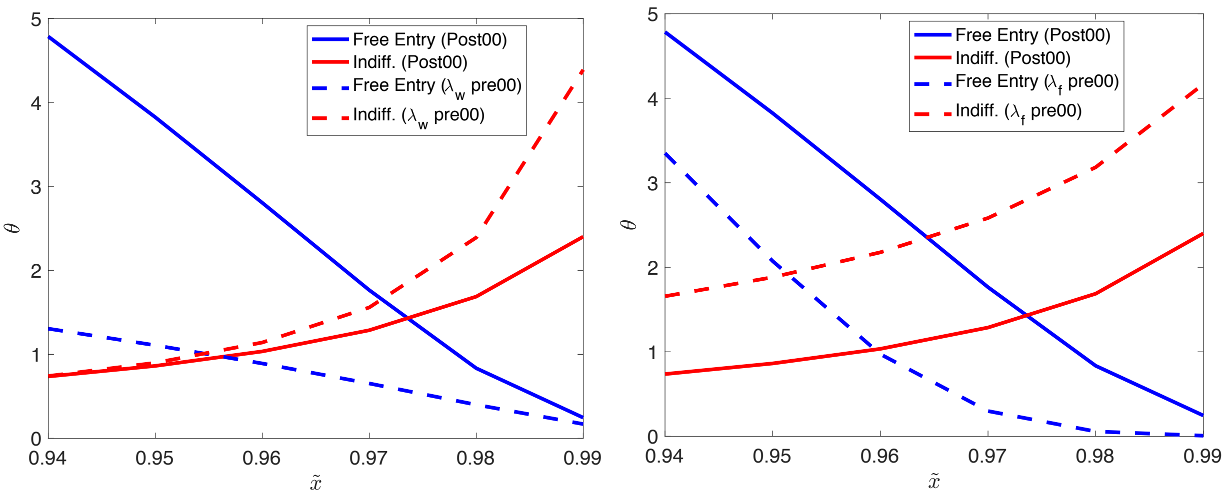
The role of worker on-the-job search: We begin first by holding λ_w at its pre-2000 value and allowing all other parameters to be updated to their post-2000 value. One argument for why the productivity-wage gap has widened is that worker on-the-job search had declined over time. A lower opportunity to climb the job ladder, i.e. lower λ_w, reduces worker's effective outside option and the pass-through of productivity to wages, widening the productivity-wage gap.

Table 3: Counterfactual: λ_w fixed at pre 2000 level

λ_w counterfactual				
	description	λ_w pre-2000	Post 2000 (all)	Percent Diff
$\widetilde{x}$	reservation productivity	0.95	0.97	-1.9
θ	labor market tightness	1.00	1.41	-29
$\lambda_f q$	firm OTJ contact rate	0.08	0.07	11
$\lambda_w p$	worker OTJ contact rate	0.09	0.02	269
job insecurity	fraction of EU that is endogenous	0.36	0.40	-10
$\lambda_f v^m/v$	fraction of vacancies firm OTJ	0.51	0.64	-20
matching efficiency	-	0.60	0.47	27
ρU	worker outside option	0.78	0.79	-0.5
Y/N	labor productivity	1.08	1.09	-0.3
mean w	average wage	0.80	0.80	-0.6
$\frac{Y/N}{\text{mean} w}$	productivity-wage gap	1.36	1.35	0.3

Table 3 shows our results. Firstly, keeping λ_w fixed at its higher pre-2000 value leads to a lower $\widetilde{x}$ and lower labor market tightness, θ. This is because when λ_w is high, firms are less able to retain workers as employed workers have a higher ease of searching on-the-job. This lower retention probability causes the free entry curve to shift leftward as depicted in Figure 8a, implying a lower entry of new jobs in the labor market. At the same time, since a higher λ_w affords workers greater ease to search while on-the-job, the workers' selectivity over reservation match quality declines, causing the worker indifference curve to shift leftward. Quantitatively, because the shift in the free entry condition is larger than the shift in the worker indifference curve, both $\widetilde{x}$ and θ are lower in equilibrium as depicted in

[39] In all our exercises, we keep χ fixed at its post 2000 value.



(a) λ_w pre 2000

(b) λ_f pre 2000

Figure 8: Equilibrium Under Our Counterfactual Exercises

Figure 8a. The lower $\widetilde{x}$ results in a larger weight on lower productivity values, and corresponds to a 0.3% lower labor productivity relative to its post 2000 counterpart.

Next, we observe that when λ_w is fixed at its pre-2000 value, the rate at which employed workers can contact vacancies, $\lambda_w p$, is higher than the rate at which matched firms can contact applicants, $\lambda_f q$. Consequently, a larger proportion of applicants are employed job-seekers who only accept a new match when productivity is higher than their current match quality. A higher proportion of employed job-seekers reduces the likelihood that a firm endogenously lays off its current worker because of successful firm on-the-job search.[40] Thus, relative to the post 2000 period benchmark, the higher λ_w reduces job insecurity by 10%. In addition, the fraction of vacancies that are firms conducting on-the-job search falls by 20% and measured matching efficiency rises by 27% relative to its post 2000 counterpart.[41]

Despite the decline in job insecurity and the rise in measured matching efficiency, the value of unemployment still falls marginally by 0.5% when λ_w is fixed at its higher pre-2000 value. This is because labor market tightness also falls by 29% relative to its post 2000 counterpart under this counterfactual exercise, making it harder for unemployed workers to contact vacancies on average. The decline in labor market tightness together with the lower $\widetilde{x}$ counteracts the gains from decreased job insecurity and higher matching efficiency, causing the effective outside option of workers and consequently, the average wage to remain roughly constant. Because both labor productivity and average wages do not vary much from their post 2000 counterpart, our counterfactual suggests that even if λ_w had not declined post 2000, the productivity-wage gap in the post 2000 period would have roughly widened by the same amount as predicted in our baseline.

[40] Unlike the case where the firm meets an unemployed job-seeker, the new match quality must be higher than the maximum of the firm's current match quality and the employed applicant's current match productivity.

[41] The fraction of filled vacancies falls because the worker now has a higher rate of searching on-the-job. When the worker re-matches with a new firm, it vacates its old job, giving rise to an unfilled vacancy. As such, the share of vacancies that are filled falls while the share of unfilled vacancies rises in this case.

The role of firm on-the-job search: We repeat the same exercise as before but now hold λ_f to its pre-2000 value and update all other parameters to their post 2000 values. When λ_f is kept at its higher pre-2000 value, firms can easily replace workers, reducing the worker's value from being employed and thus also their value of unemployment. As such, Figure 8b shows that when λ_f is kept at its pre-2000 value, the workers' indifference curve shifts leftward as workers become less selective over their reservation match quality. At the same time, the higher opportunity of re-matching with applicants while having a filled position causes firms to also be less selective over their reservation match quality and for the free entry curve to shift leftward. Overall, the higher pre-2000 λ_f causes reservation match quality to fall and labor market tightness to increase as depicted in Figure 8b.

Table 4: Counterfactual: λ_f fixed at pre 2000 level

λ_f counterfactual				
	description	λ_f pre-2000	Post 2000 (all)	Percent Diff
$\widetilde{x}$	reservation productivity	0.95	0.97	-2.3
θ	labor market tightness	1.92	1.41	36
$\lambda_f q$	firm OTJ contact rate	0.14	0.07	111
$\lambda_w p$	worker OTJ contact rate	0.03	0.02	23
job insecurity	fraction of EU that is endogenous	0.53	0.40	34
$\lambda_f v^m/v$	fraction of vacancies firm OTJ	0.82	0.64	28
matching efficiency	-	0.35	0.47	-27
ρU	worker outside option	0.78	0.79	-0.2
Y/N	labor productivity	1.12	1.09	3.2
mean w	average wage	0.80	0.80	-0.6
$\frac{Y/N}{\text{mean} w}$	productivity-wage gap	1.39	1.35	2.5

With λ_f fixed at its pre-2000 value, the rate at which firms can conduct on-the-job search is close to five times larger than the rate at which workers can search on-the-job ($\lambda_f \times q = 0.14$ vs $\lambda_w \times p = 0.03$). Because matched firms have a higher rate of contacting applicants, and because a larger share of applicants are unemployed[42] this raises the likelihood that the firm is able to replace its incumbent whenever it meets an applicant who draws a productivity greater than that of its current worker's match quality. This in turn raises the firm's ability to climb into better matches, contributing to a 3.2% higher labor productivity.

The higher contact rate of the matched firm also raises job insecurity and the share of vacancies that are firms searching on-the-job, $\lambda_f v^m/v$, by 34 and 28 percent relative to its post 2000 counterpart. This higher $\lambda_f v^m/v$ in turn contributes to the 27% decline in measured matching efficiency. The rise in job insecurity and the decline in measured matching efficiency work towards lowering workers' unemployment value. In the equilibrium, the value of unemployment is lower by 0.2% despite a higher labor market tightness. Because labor productivity is higher while the average wage is largely unchanged, a wider productivity-wage gap emerges. Intuitively, the firm's higher ability to search on-the-job reduces

[42] In our counterfactual exercise, the share of applicants who are unemployed is 0.84. In contrast, the share of applicants who are unemployed in our post 2000 benchmark is 0.78.

the worker's effective outside option relative to that of the firm, allowing firms to pass-through a smaller share of productivity to wages, widening the productivity-wage gap.

Our results suggest that had λ_f not also fallen in the post 2000 period, the divergence in the productivity-wage gap would have been even higher. Thus, we conclude that the changing ease of firm on-the-job search plays an important role in affecting the magnitude of the productivity-wage gap.

5.2.1 Robustness

Recent work by Fujita et al. (2019) argues that employment-to-employment (EE) transitions have not actually declined post 2000 but are actually an artefact of an increase in missing answers surrounding the question on whether the individual was still with the same employer as the previous interview month.[43] To address the possibility that employment-to-employment transitions did not decline, we repeat our calibration exercise but make the targeted EE share in the post 2000 period equal to that observed in the data pre-2000. Appendix D contains our results. Importantly, relative to the post 2000 period where the EE share was allowed to fall, we find that fixing the EE share implies a higher λ_f and λ_w, and a lower χ for the post 2000 period. Because λ_f, λ_w and χ are all changing in our model recalibrated to match a fixed EE share, we isolate the effects of λ_w and λ_f separately by repeating the counterfactual exercises as described in Section 5.1.1. Table 9 in Appendix D shows our results. We find the same results as before, keeping λ_w fixed at its pre-2000 level does little to change the productivity-wage gap while keeping λ_f fixed at its pre-2000 value would have raised the productivity-wage gap by 2%. As such, our model's main finding that the productivity-wage gap depends on firms' ability to conduct on-the-job search relative to workers continues to hold even when we re-calibrate the model to match a constant EE hiring share across time periods.

6 Discussion

Changes in λ_f and λ_w Thus far, we have treated λ_f and λ_w as reduced form parameters that capture the ease of firm and worker on-the-job search. One can think of these parameters as stand-ins for how the contractual term-length of a job has changed over time. The recent rise in non-compete contracts which acts toward reducing the ease of worker on-the-job search and would be captured in our model by a falling λ_w over time.[44] At the same time, a greater use of at-will employment contracts or contract workers by firms would imply higher job insecurity to workers. Recent work by Katz and Krueger (2016) shows that the percentage of individuals employed in non-standard work arrangements grew by about 5.1 percentage points between 2005 and 2015.[45] Notably, contracted workers represented the fastest growing segment amongst non-standard work arrangements. The higher job insecurity implied by the greater usage of contract workers would in our model be captured by a higher λ_f relative to λ_w as job insecurity is defined as the fraction of exits into unemployment that are triggered by a firm replacing a worker. Implicitly, the reciprocal of λ_f captures the term-length where the firm is restricted from firing

[43] This does not mean that the hiring rate has not declined since using data from the QWI, we show that the hiring rate i.e. hires over employment was declining over time.

[44] Recent work by Shi (2018) shows that 64% of executives in public firms are subject to non-compete contracts.

[45] Non-standard work arrangements include individuals who are contract workers, temporary help agency workers, on-call workers and independent contractors.

the worker.[46] Hence, when λ_f is higher relative to λ_w, the worker is more likely to be displaced into unemployment for reasons of firm on-the-job search. Thus, a higher λ_f relative to λ_w can be thought of as capturing the increased ease of firm on-the-job search through the higher usage of contract workers as well as the depressed ability of workers to conduct on-the-job search because of the coincident rise of non-compete contracts.

Wage Determination We assume that wages are determined via Nash Bargaining and that firms (workers) must leave their incumbent workers (firms) prior to bargaining with their new applicants (employers). The results in our paper would not qualitatively change if we had instead assumed a different wage determination protocol such as that of sequential auctions as in Cahuc et al. (2006). Note that in this case, matched workers leave their incumbent firm whenever the vacancy contacted offers the worker a value larger than what its incumbent firm can offer to retain her. Similarly, matched firms leave their incumbent worker whenever the applicant they meet gives the firm a larger surplus than what it can observe with its current worker. For matches where the vacancy contacted cannot counter the incumbent firm's highest offer, the offer from the vacancy contacted can still be used to bump up the worker's outside option and to renegotiate wages upward. In the same vein, when the firm does on-the-job search and contacts an applicant who cannot counter its incumbent worker's highest offer to retain the firm, the potential match with the new applicant can be used to renegotiate the incumbent worker's wage down. As such, the prospect of firm on-the-job search still promotes a widening productivity-wage gap. In our Nash bargaining protocol, this works through lowering workers' outside options. In the sequential auctions framework, it additionally comes through renegotiating incumbent workers' wages down. The incidence of replacement hiring may differ under the two different wage setting protocols, since replacement hires would only occur in the sequential auctions framework when the new match surplus is greater than each agent's current surplus from their existing matches.

We have also assumed in our model that workers and firms must leave their current partners before forming a new match and moving to the bargaining stage. We do this because the bargaining set becomes non-convex if agents are allowed to use their current matched value as their outside option, and the axioms under which Nash Bargaining is defined do not hold.[47] Our results, however, would still apply if we considered a model where wages were determined via an exogenous surplus splitting rule. This implies that the wages delivered by the firm require it to pay workers their outside option and a fixed share of the surplus. In addition, we now allow matched workers and firms to use their current matched value as their outside option whenever they contact a new vacancy or applicant. In this environment, our result that a higher incidence of firm on-the-job search would lead to a larger productivity-wage gap still applies. Even if workers could use their current employment values as their outside option, a higher incidence of firm on-the-job search reduces the worker's employment spells and hence their employment value. The lower employment value/outside option in turn depresses the wage the worker takes home. Thus, qualitatively, our results do not depend on the wage-setting rule we have used.

[46] In other words, if a firm commits to hiring a worker for 2 years, he cannot replace the worker within the two years although he can do so if he continues to hire the worker after the first 2 years.

[47] See Shimer (2006) for more information.

Vacancy duration and the relevant measure of market tightness The standard DMP model assumes that vacancies expire instantaneously, implying that only the current flow of vacancies are relevant for computing market tightness. In contrast, we argue that our assumption of long-lived job positions better accords with how the data on job openings is collected. In particular, the Bureau of Labor Statistics (BLS) which conducts the monthly JOLTS states that the information it collects on job openings are "a stock,or point-in-time, measurement for the last business day of each month".[48] Further, our measure of vacancies is *not* inconsistent with the JOLTS definition of a job-opening. Specifically, JOLTS requires a job opening to satisfy three criteria: 1) a position exists, 2) work can start in 30 days, and 3) the firm is actively recruiting where active recruiting implies that the firm has undertaken "steps to fill a position". Firms who conduct on-the-job search in our model satisfy these three conditions. Our model calls to attention that the proportion of unfilled vacancies in addition to labor market tightness are important for rationalizing job-finding rates.

Restructuring One issue with the data on replacement hiring is that the QWI does not have information by occupation. As such, one concern may be that firms who re-structure fire/lay-off their current workers whose skills may not be suited for tasks under their new production process. In other words, one cannot observe within the data if the firm, when conducting a replacement hire, is hiring a new applicant for the same position. We argue that while we cannot directly observe this, our model does have implications for how wages would grow. To see this, consider the following example. Suppose the firm used to hire janitors but then decided to re-structure and fire all its janitors and replace them with engineers. Here, the growth rate in wages would be given by the percent difference between the average wage of engineers and the average wage of janitors. In this case, such re-structuring suggests a large change in wage growth as firms swap out their work-force with workers who perhaps better complement their new production processes. In contrast, our model has a different implication for how wages would grow if instead firm on-the-job search is active and firms were replacing their incumbents with higher quality applicants. In particular, we showed that when firm on-the-job search is active, it leads to a gentler slope of the wage function with respect to changes in productivity. Thus, our model with its implications on how wages grow with changes in productivity can be used to distinguish whether restructuring or firm on-the-job search is active.

Additional hiring costs Our paper also assumes a fixed cost of creating job positions and abstracts from other types of hiring costs. Inclusion of a hiring cost paid at the time of matching with a worker would not qualitatively change the predictions of our model. Its inclusion would only serve to raise hiring thresholds. So long as the replacement hiring share increases, this will still drive a larger productivity-wage gap. Moreover, the one time fixed vacancy posting cost χ, rather than the standard flow cost of a vacancy, can be interpreted as the cost of maintaining a human resources department for the duration of the job position and therefore subsumes the additional costs that could have been incurred at the time of matching.

[48]See the JOLTS chapter of the "BLS Handbook of Methods". https://www.bls.gov/opub/hom/pdf/homch18.pdf

7 Conclusion

We document that the replacement hiring share in the US has risen over time alongside a widening productivity wage-gap. We develop a model that incorporates both worker and firm on-the-job search and examine the implications that worker vs. firm on-the-job search has for productivity and wages. We find that, holding all else constant, both firm and worker on-the-job search cause productivity to increase as workers and firms climb the ladder for better matches. However, the extent to which productivity is passed-through to wages depends critically on whether worker vs. firm on-the-job search is more prevalent. When firms can search on-the-job search at a higher rate than workers, the effective outside option of the firm is elevated relative to workers, allowing them to pass-through a smaller share of productivity to wages. All this encourages a wider productivity-wage gap. Quantitatively, our model can explain the observed widening of the productivity wage gap. Furthermore, we show that if the firm's ability to conduct on-the-job search stayed at its pre-2000 levels, the productivity-wage gap would have been even higher by about 2.5%.

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Appendix

A Additional Data Work

A.1 Examining Replacement Hires

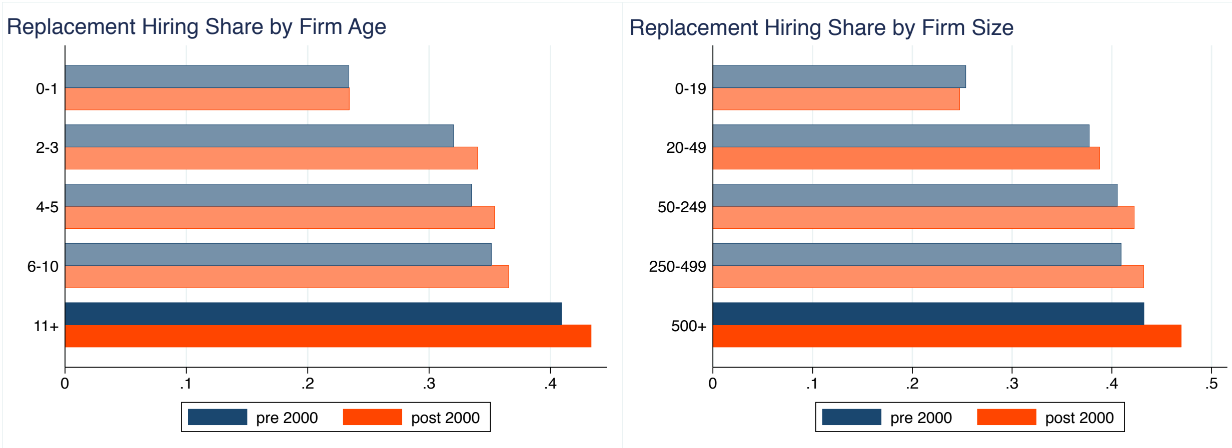
Figure 9a plots the replacement hiring rate against the hiring rate over time. The hiring rate is defined as the ratio of total hires to total employed while the replacement hiring rate is given by the ratio of replacement hires to total employed. Figure 9a highlights that higher replacement hiring share is a result of hires declining faster than replacement hires.

(a) Replacement vs. total hiring rate

A.2 Shift Share Analyses

To see if compositional changes in sectors are the primary factor behind the rise in the replacement hiring share, we conduct a shift-share analysis. In examining whether compositional changes were behind the rise in the replacement hiring share, we cut the data separately by firm age, firm size, by industry and by worker education. Data by firm age and size, and by worker education are only reported for private firms in the QWI. We follow the firm age and size, and worker education categories as provided by the QWI. For the shift share analysis done at the industry level, we use information available at the 2 digit NAICS level.

Because the divergence in the productivity-wage gap largely occurred after the 2000s, we divide the periods into pre-2000 and post 2000, and take the mean of the replacement hiring share in these two periods. We weight each firm age/size/industry/worker education category by their average employment share. Across the two time periods, the average replacement hiring share rose by about 3 percentage points. To assess how much of the increase in the replacement hiring share can be accounted for by compositional changes (“between”) vs. by just an increase within each firm age/size/industry/worker education category, we use the following decomposition:



(a) By Firm Age

(b) By Firm Size

Figure 10: Replacement Hiring Shares by Firm Age and Size

$$\Delta\text{Replacement Hiring Share} = \sum_{it} \Delta \left(rr_{it} \frac{emp_{it}}{emp_t} \right) \approx \underbrace{\sum_{it} \widehat{rr} \Delta \left(\frac{emp_{it}}{emp_t} \right)}_{\text{between}} + \underbrace{\sum_{it} \Delta rr_{it} \left(\widehat{\frac{emp_i}{emp}} \right)}_{\text{within}}$$

where Δ refers to the change, rr_{it} denotes the replacement hiring share of firm age/size/industry category i in period t, and emp_{it} denotes the employment in category i in period t. Terms with a "hat" denote averages. Table 5 shows the results from our shift share analysis:

Table 5: Shift-Share Analysis

Percent Explained by	Between	Within
By Firm Age	19.9%	80.1%
By Firm Size	26.7%	73.4%
By Industry	-0.0%	100%
By Worker Education	-2%	102%

A few things are noteworthy. Firstly, across firm age and size, we find that large and older firms were more likely to conduct replacement hiring. This is consistent with the notion that once firms reach their optimal firm size, they conduct replacement hires either for the purposes of finding a better worker or for the purposes of re-filling a position. Figures 10a and 10b show how replacement hiring shares vary across firm age and size for the two time periods.

Given then large and older firms are more likely to conduct replacement hiring, Our shift-share analyses reveal that compositional changes at the firm age and firm size level explain close to a quarter of the changes in the replacement hiring share. While compositional changes at the firm age and firm size level contribute to the increase in the replacement hiring share, Table 5 highlights that a significant bulk of the increase in the replacement hiring share stems from changes within each firm age/size/industry/worker education category. This result is particularly stark at the industry and the worker education level. Focusing first on industries, Figure 11 shows that this is largely because the ranking of industries by employment share has not significantly changed, with the top 5 industries in the pre 2000 and post 2000 being exactly the same. In fact, our shift share analysis actually suggests that the change in industry composition actually contributes towards reducing the replacement hiring share by 0.005 points as opposed to increasing it.

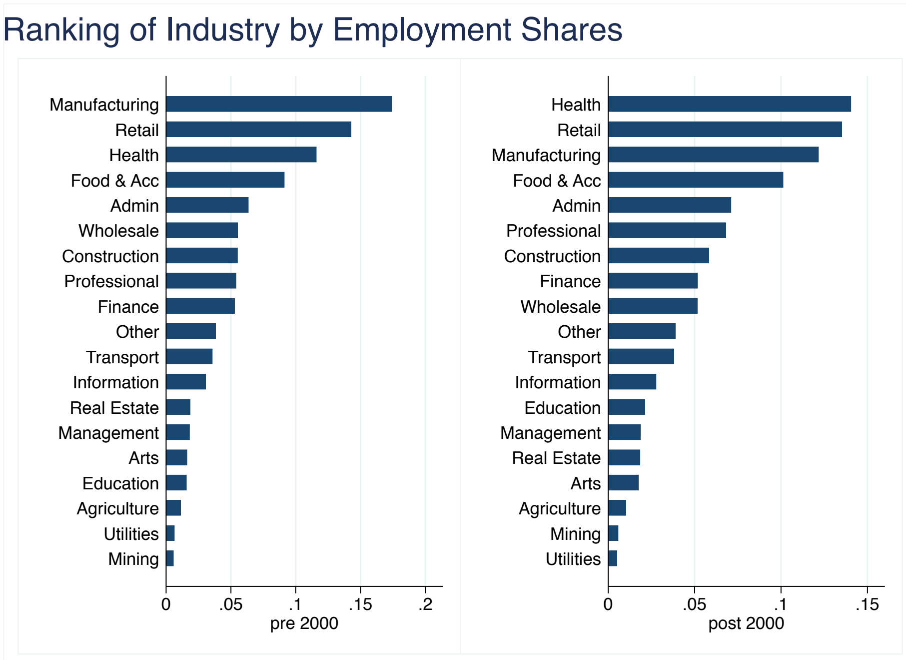


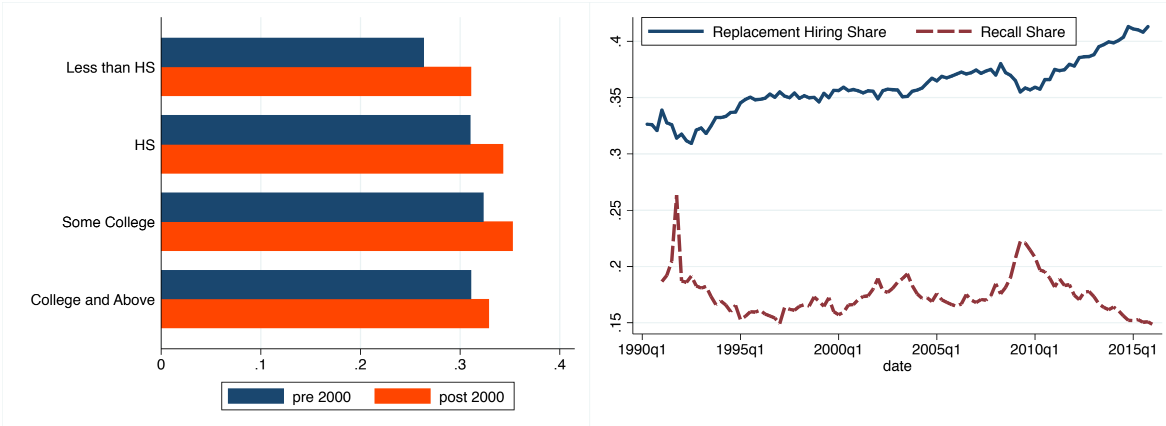
Figure 11: Ranking of industries roughly the same

Looking next at the changes by worker composition, Figure 12a shows that the replacement hiring share increased within each worker education category. Note that in these measures, the information presented here is that for a hire at a firm, the firm knows the worker's education level. Thus, the following replacement shares represent the fraction of hires for individuals with a certain education level that are replacement hires. We, however, do not know if the replacement hire recorded was for a position that required the same education level. Across the two time periods, we find that changes in the between component would have reduced the replacement hiring share by 2%, highlighting that the increase in the replacement hiring share *within* each education category explains the overall rise.

A.3 Recalls vs. Replacement Hires

Finally, we examine if the replacement hiring share could be explained by recalls. In reality, some replacement hires may be recalls. To see this, consider a firm who had a worker A in period $t-2$, but worker A was temporarily laid off in period $t-1$, while worker B was hired in period $t-1$. Worker B then left the firm in period t and worker A was recalled in period t. In this example, the net employment change at the firm in periods $t-1$ and t were zero, there was one replacement hire each in period $t-1$ and t, and the replacement hire in period t was also a recall hire. This example illustrates how some replacement hires can be recall hires.

The QWI also provides information on the number of recall hires at an establishment. A recall hire



(a) Replacement Hiring by Education (b) Recall vs. Replacement Hiring Share

Figure 12: Replacement Hiring Share by Education and Replacement Hiring Share vs. Recall Share

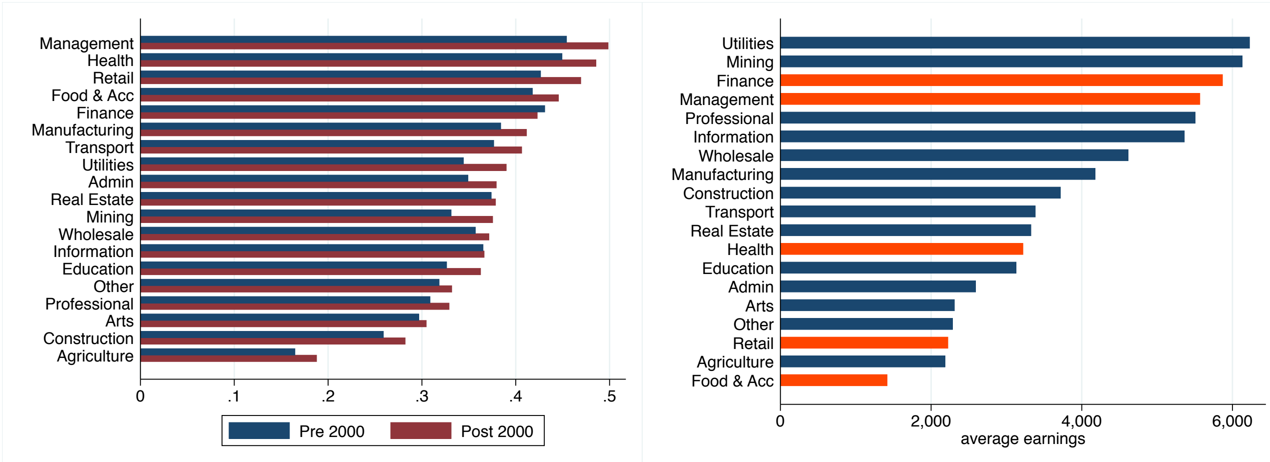
Table 6: An Example of Recalls and Replacement Hires

	$t-2$	$t-1$	t
A	1	0	1
B	0	1	0
Replacement hires at t			2
Recall hires at t			1

is recorded whenever an individual i has earnings at establishment j in period t, had no earnings at j in period $t-1$ but recorded earnings at j in any previous period $t-2, t-3$ or $t-4$. We compute the recall share of hires as the ratio of recall hires to total hires and plot the recall hiring share against the replacement hiring share in Figure 12b. Figure 12b shows that while the recall hiring share rose during the Great Recession (an observation also documented by Fujita and Moscarini (2017) using data from the Survey Income and Program Participation (SIPP), its rise was not permanent. In the aftermath of the Great Recession, the recall hiring share fell suggesting that the rise in the replacement hiring share is not explained by an increase in recall hiring.

A.4 Replacement Hiring Across Industries and Earnings

To examine if industries with low average earnings are responsible for the high replacement hiring share, we use the industry aggregated information from the QWI on the average monthly earnings of employees who worked with the same firm throughout the quarter and compare whether the industries who have high replacement hiring share also have low average earnings. Figure 13a highlights that most industries observed an increase in their replacement hiring share post 2000 and that the top five industries which have a high share of replacement hiring are Management of Companies and Enterprises, Healthcare and Social Assistance, Retail Trade, Accommodation and Food Services and Finance and Insurance. Figure 13b highlights that these top five industries by replacement hiring share (shaded in orange) are not exclusively low-paying industries. In fact, both Management of Companies and Enterprises and Finance and Insurance appear at the upper end of the average earnings distribution.



(a) Replacement Hiring Share by Industry (b) Average Monthly Earnings by Industry

Figure 13: Replacement Hiring and Earnings by Industry

B Surplus, Wages and Distribution of Matched Firm-worker Pairs

Recruiting firm-worker pair Subtract (4) from (1) and rearrange to get the firm's gain to matching:

$$(\rho + \delta + s + q^*(x) + p^*(x)) [J(x) - \chi] = x - w(x) - \left[(\rho + \delta)\chi - \widehat{R}(x)\right] \tag{28}$$

where

$$\begin{aligned} \widehat{R}(x) &= \lambda_f q \frac{u}{\ell} \int_x^{\overline{x}} [J(y) - J^u] \, d\Pi(y) \\ &+ \lambda_f q \frac{\lambda_w v^m}{\ell} \left\{ \int_x^{\overline{x}} [J(y) - J^u] \, d\Pi(y) F(x) + \int_x^{\overline{x}} \int_\epsilon^{\overline{x}} [J(y) - J^u] \, d\Pi(y) \, dF(\epsilon) \right\} \end{aligned}$$

Observe that $(\rho + \delta)\chi - \widehat{R}(x)$ represents the firm's effective outside option. The firm's effective outside option or relative gain from continuing to search as an unfilled vacancy is the higher meeting rate q and all the possible matches from $\widetilde{x}$ to x. Next, subtracting ρU from both sides of (8) yields us the worker's gain to matching:

$$(\rho + \delta + s + q^*(x) + p^*(x)) [W(x) - U] = w(x) - \left[\rho U - \widehat{H}(x)\right] \tag{29}$$

where

$$\begin{aligned} \widehat{H}(x) &= \lambda_w p \frac{v^u}{v} \int_x^{\overline{x}} [W(y) - U] \, d\Pi(y) \\ &+ \lambda_w p \frac{\lambda_f v^m}{v} \left\{ \int_x^{\overline{x}} [W(y) - U] \, d\Pi(y) F(x) + \int_x^{\overline{x}} \int_\epsilon^{\overline{x}} [W(y) - U] \, d\Pi(y) \, dF(\epsilon) \right\} \end{aligned}$$

Observe that $\rho U - \widehat{H}(x)$ is the worker's effective outside option. It represents all the foregone potential matches from $\widetilde{x}$ to x the worker could have undertaken if she chose to continue to search as an unemployed worker.

Adding the two expressions and re-arranging gives us surplus:

$$\begin{aligned} \varrho(x) S(x) &= x - \rho U - (\rho+\delta)\chi \\ &+ \left[q(1-\eta)\frac{\lambda_f u}{\ell} + p\eta \frac{\lambda_w v^u}{v}\right] \int_x^{\overline{x}} S(y)\, d\Pi(y) \\ &+ q\frac{\lambda_f \lambda_w v^m}{\ell} \left\{ \int_x^{\overline{x}} S(y)\, d\Pi(y)\, F(x) + \int_x^{\overline{x}} \int_\epsilon^{\overline{x}} S(y)\, d\Pi(y)\, dF(\epsilon) \right\} \end{aligned} \tag{30}$$

where $S(x) = J(x) - J^u + W(x) - U$ and $\varrho(x) = \rho + \delta + s + p^*(x) + q^*(x)$. Next using the Nash Bargaining solution (12), the free entry condition (i.e. equation 4) can be re-written as:

$$(\rho+\delta)\chi = (1-\eta) q \left[\frac{u}{\ell} \int_{\widetilde{x}}^{\overline{x}} S(y)\, d\Pi(y) + \lambda_w \frac{1-u}{\ell} \int_{\widetilde{x}}^{\overline{x}} \int_\epsilon^{\overline{x}} S(y)\, d\Pi(y)\, dF(\epsilon) \right]$$

Evaluating 30 at $\widetilde{x}$ and plugging for free-entry, we get equation 18.

B.1 General Wage Form

Note from the worker's gain to matching, we have:

$$(\rho + \delta + s + q^*(x) + p^*(x)) \eta S(x) = w(x) - [\rho U - \widehat{H}(x)]$$

Using the form of $\widehat{H}(x)$ and making $w(x)$ the subject of the equation, we have:

$$\begin{aligned} w(x) &= \eta x + (1-\eta)\rho U - \eta(\rho+\delta)\chi \\ &+ (1-\eta)\eta \left[\lambda_f q \frac{u}{\ell} - \lambda_w p \frac{v^u}{v}\right] \int_x^{\overline{x}} S(y)\, d\Pi(y) \end{aligned} \tag{31}$$

B.2 Distribution of productivity

Focusing on the measure of matched firm-worker pairs with match quality less than or equals to x and dividing everywhere by $v^m = (1-u)$, we get:

$$\begin{aligned} \left[q\frac{v^u}{(1-u)}\frac{u}{\ell}\right] [\Pi(x) - \Pi(\widetilde{x})] &= (s+\delta) F(x) + qF(x)[1-\Pi(x)] \left[\lambda_f \frac{u}{\ell} + \lambda_w \frac{v^u}{\ell}\right] \\ &+ 2\lambda_f q F(x) \frac{\lambda_w F(x)(1-u)}{\ell} [1-\Pi(x)] \\ &+ 2\lambda_f q F(x) \frac{\lambda_w (1-u)}{\ell} \int_x^{\overline{x}} [1-\Pi(\epsilon)] f(\epsilon)\, d\epsilon \\ &+ \lambda_f q \frac{\lambda_w (1-u)}{\ell} \int_{\widetilde{x}}^{x} \int_z^{x} [\Pi(x) - \Pi(\epsilon)] f(\epsilon) f(z)\, d\epsilon dz \\ &+ \lambda_f q \frac{\lambda_w (1-u)}{\ell} \int_{\widetilde{x}}^{x} [\Pi(x) - \Pi(z)] F(z) f(z)\, dz \end{aligned} \tag{32}$$

C Comparative Statics

C.1 Worker On-the-job Search Only

C.1.1 Distribution of Matched Firm-Worker Pairs

Setting $\lambda_f = 0$, and using equation 32, we can show that the cumulative distribution of firm-worker pairs with match quality less than or equals to x becomes:

$$F(x) = \left[q\frac{v^u}{(1-u)}\frac{u}{\ell}\right]\frac{[\Pi(x) - \Pi(\widetilde{x})]}{(s+\delta+p\lambda_w[1-\Pi(x)])}$$

From the law of motion for the unemployed, we have:

$$q\frac{v^u}{1-u}\frac{u}{\ell} = \frac{s+\delta}{[1-\Pi(\widetilde{x})]}$$

plugging this into $F(x)$, we get:

$$F(x) = \frac{s+\delta}{(s+\delta+p\lambda_w[1-\Pi(x)])}\frac{\Pi(x)-\Pi(\widetilde{x})}{1-\Pi(\widetilde{x})}$$

C.1.2 Surplus and Pass-through

In the case where only workers can search on-the-job, i.e. $\lambda_f = 0$, the only vacancies workers can contact are unfilled vacancies, i.e. $v = v^u$. In this case, surplus of a match as given by equation 13 collapses to equation 20 which we replicate below for the readers convenience:

$$(\rho+\delta+s+\lambda_w p[1-\Pi(x)])S(x) = x - (\rho+\delta)J^u - [\rho U - \widehat{H}(x)]$$

Evaluating the above equation at $\widetilde{x}$ allows us to define the worker's effective outside option to be as expressed as in equation 21, again replicated below for ease of convenience.

$$\rho U - \widehat{H}(x) = \widetilde{x} - (\rho+\delta)J^u + \lambda_w p\eta\int_{\widetilde{x}}^{x} S(y)d\Pi(y)$$

Holding $\widetilde{x}$ and θ constant, the worker's effective outside option is increasing in λ_w because the worker must be increasing compensated for foregone opportunities when λ_w is higher. Further, equation 4 shows that as λ_w rises, the composition of job-seekers tilts towards that of employed job-seekers. This worsens the firm's value of an unfilled vacancy holding all else constant since the firm must now draw a match quality higher than the employed worker's incumbent firm before the worker agrees to form a new match. A higher λ_w, holding all else constant, reduces matching efficiency for an unfilled vacancy and thus lowers its value.

From equation 31 when $\lambda_f = 0$, the wage of type x also becomes

$$w(x) = \eta x + (1-\eta)\rho U - (1-\eta)\eta\lambda_w p\int_{x}^{\overline{x}} S(y)d\Pi(y) - \eta(\rho+\delta)J^u$$

Pass-through is then found by differentiating wages with respect to x which gives us back equation 22:

$$w'(x) = \eta + (1-\eta)\,\eta\lambda_w p\pi(x)S\,(x)$$

At this point it is useful to note that taking the derivative of surplus, i.e. equation 20 with respect to x, we get:

$$-\frac{(1-\eta)\,\lambda_w p\pi\,(x)\,S\,(x)}{\rho+s+\delta+\lambda_w p\,[1-\Pi\,(x)]}+\frac{dS\,(x)}{dx}=\frac{1}{\rho+s+\delta+\lambda_w p\,[1-\Pi\,(x)]}$$

The above is an ODE and surplus can also be expressed as:

$$S(x)=\frac{1}{(\rho+s+\delta+\lambda_w p[1-\Pi(x)])^{1-\eta}}\int_{\widetilde{x}}^{x}\frac{1}{(\rho+s+\delta+\lambda_w p[1-\Pi(y)])^{\eta}}dy \tag{33}$$

Plugging equation 33 into 22, one can show that the amount of pass-through of productivity to wages is positively affected by the ease of worker on-the-job search, λ_w, ceteris paribus. Pass-through can be re-stated as:

$$w'(x) \quad = \quad \eta+(1-\eta)\eta\frac{1}{(\frac{\rho+s+\delta}{\lambda_w p}+\widehat{\Pi}(x))^{1-\eta}}\int_{\widetilde{x}}^{x}\frac{1}{(\frac{\rho+s+\delta}{\lambda_w p}+\widehat{\Pi}(y))^{\eta}}dy\pi(x) \tag{34}$$

where $\widehat{\Pi}(x)=1-\Pi(x)$. The equation above shows that $w'(x)$ is rising in λ_w.

C.2 Firm On-the-job Search Only

C.2.1 Distribution of Matched Firm-Worker Pairs

Shutting down worker on-the-job search, $\lambda_w=0$, one can show that equation 32 becomes:

$$F(x)=q\frac{v^u}{1-u}\frac{[\Pi(x)-\Pi(\widetilde{x})]}{s+\delta+\lambda_f q[1-\Pi(x)]}$$

At the same time, the law of motion for the unemployed becomes:

$$q\frac{v^u}{1-u}=\frac{s+\delta}{1-\Pi(\widetilde{x})}$$

Plugging the above into $F(x)$, we get that the distribution of matched firm-worker pairs with match quality $\leq x$ is given by:

$$F\,(x)=\left(\frac{s+\delta}{s+\delta+\lambda_f q\,[1-\Pi\,(x)]}\right)\frac{\Pi\,(x)-\Pi\,(\widetilde{x})}{1-\Pi\,(\widetilde{x})}$$

C.2.2 Surplus and Pass-through

In this case when $\lambda_w=0$, we have $u=\ell$. Equation 30 then simplifies to equation 24, replicated below for ease of convenience:

$$(\rho+\delta+s+\lambda_f q[1-\Pi(x)])\,S(x)=x-\rho U-[(\rho+\delta)J^u-\widehat{R}(x)]$$

Evaluating the above at $\widetilde{x}$, we arrive at the effective outside option of the firm, equation 25, replicated again below for ease of reference:

$$(\rho+\delta)J^u - \widehat{R}(x) = \widetilde{x} - \rho U + \lambda_f q(1-\eta)\int_{\widetilde{x}}^{x} S(y)d\Pi(y)$$

Turning to wages and using equation 31, under $\lambda_w = 0$, we get

$$w(x) = \eta x + (1-\eta)\rho U + (1-\eta)\eta\lambda_f q\int_{x}^{\overline{x}} S(y)\,d\Pi(y) - \eta(\rho+\delta)J^u$$

Differentiating $w(x)$ with respect to x, we arrive at our pass-through equation, 26:

$$w'(x) = \eta - \eta(1-\eta)\lambda_f q S(x)\pi(x)$$

Diffentiating 24 with respect to x, we have:

$$-\frac{\eta\lambda_f q\pi(x)}{(\rho+\delta+s+\lambda_f q[1-\Pi(x)])}S(x) + S'(x) = \frac{1}{(\rho+\delta+s+\lambda_f q[1-\Pi(x)])}$$

The above is an ODE which upon solving, we arrive at 35.

$$S(x) = \frac{1}{(\rho+s+\delta+\lambda_f q[1-\Pi(x)])^{\eta}}\int_{\widetilde{x}}^{x}\frac{1}{(\rho+s+\delta+\lambda_f q[1-\Pi(y)])^{1-\eta}}dy \tag{35}$$

Plugging 35 into $w'(x)$, we get equation 36.

$$w'(x) = \eta - \eta(1-\eta)\pi(x)\frac{1}{(\frac{\rho+s+\delta}{\lambda_f q}+\widehat{\Pi}(x))^{\eta}}\int_{\widetilde{x}}^{x}\frac{1}{(\frac{\rho+s+\delta}{\lambda_f q}+\widehat{\Pi}(y))^{1-\eta}}dy \tag{36}$$

The equation above shows that $w'(x)$ is declining in λ_f.

D Recalibrated Model: Fixed EE Share

D.1 Robustness Check: Fixed EE share

Following Fujita et al. (2019) who argue that EE rates have not actually declined over time, we fix our targeted EE share in the post 2000 period to be equal to the EE share in the pre-2000 period. Table 7 shows our calibration results from this exercise. Notably, fixing the EE share elevates both the level of λ_f and λ_w relative to what we observed in our benchmark post 2000 calibration, while χ is actually lower.

Further, Table 8 shows how large the productivity-wage gap would be if we had instead calibrated the model to target a constant EE hiring share across the two time periods. While the productivity-wage gap is narrower by 1.4% in this re-calibrated version of the model, it should be noted that all parameters changed under this re-calibration, making it hard to pinpoint the contribution of worker on-the-job search.

As such, Table 9 repeats the counterfactual exercises of Section 5.1.1 and demonstrates the effect

Table 7: Model Parameters, Fixed EE share

Calibrated Parameters (post-2000,Fixed EE share)			
Parameter	Value	Quarterly Targets	Model Moment
λ_f	0.097	replacement hiring share of 0.38	0.36
λ_w	0.057	EE hiring share of 0.48	0.46
b	0.574	70% UI ratio	0.70
s	2.4e-8	exit rate of 0.070	0.069
σ_x	0.027	unemployment rate of 0.061	0.061
χ	7.009	residual from free-entry eqn	

Table 8: Counterfactual: EE share fixed at pre 2000 level

Fixed EE share counterfactual				
	description	EE share fixed	Post 2000	Percent Diff
$\widetilde{x}$	reservation productivity	0.96	0.97	-1.09
θ	labor market tightness	1.39	1.41	-0.91
$\lambda_f q$	firm OTJ contact rate	0.09	0.07	31.28
$\lambda_w p$	worker OTJ contact rate	0.07	0.02	215.06
job insecurity	fraction of EU that is endogenous	0.41	0.40	2.0
$\lambda_f v^m/v$	fraction of vacancies firm OTJ	0.61	0.64	-5.83
matching efficiency	-	0.50	0.47	106.51
ρU	worker outside option	0.80	0.79	1.98
Y/N	labor productivity	1.09	1.09	0.56
mean w	average wage	0.82	0.80	2.01
$\frac{Y/N}{\text{mean} w}$	productivity-wage gap	1.33	1.35	-1.43

of allowing workers vs. firms to have a higher opportunity of conducting on-the-job search. As before we find that if λ_f was higher, the productivity wage gap would have been larger whereas little changed under a higher λ_w.

Table 9: Counterfactual: EE share fixed at pre 2000 level

Role of λ_w vs. λ_f (Fixed EE share)				
	description	λ_w	λ_f	Post 2000
$\widetilde{x}$	reservation productivity	0.96	0.95	0.96
θ	labor market tightness	1.16	1.63	1.39
$\lambda_f q$	firm OTJ contact rate	0.09	0.15	0.09
$\lambda_w p$	worker OTJ contact rate	0.09	0.08	0.07
job insecurity	fraction of EU that is endogenous	0.39	0.51	0.41
$\lambda_f v^m/v$	fraction of vacancies firm OTJ	0.56	0.75	0.61
matching efficiency	-	0.55	0.40	0.50
ρU	worker outside option	0.80	0.80	0.80
Y/N	labor productivity	1.09	1.12	1.09
mean w	average wage	0.82	0.82	0.82
$\frac{Y/N}{\text{mean} w}$	productivity-wage gap	1.34	1.36	1.33