Slow Recoveries and Unemployment Traps: Monetary Policy in a Time of Hysteresis*

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Abstract

We analyse monetary policy in a model where temporary shocks can permanently scar the economy's productive capacity. Workers lose skill while unemployed and are costly to retrain, generating multiple steady-state unemployment rates. Following a large shock, unless monetary policy acts aggressively and quickly enough to prevent a significant rise in unemployment, hiring falls to a point where the economy recovers slowly at best – at worst, it falls into a permanent unemployment trap. Monetary policy can only avoid these outcomes if it commits in a timely manner to more accommodative policy in the future. Timely commitment is essential as the effectiveness of monetary policy is state dependent: once the recession has left substantial scars, monetary policy cannot speed up a slow recovery, or escape from an unemployment trap.

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1 Introduction

In the aftermath of the global financial crisis, economic activity remained subdued, suggesting that the world economy may have settled on a lower growth trajectory than the one prevailing before 2007. Some observers have attributed this sluggish growth to permanent, exogenous structural changes - either permanently lower productivity growth (Gordon, 2015) or *secular stagnation*.[1] An alternative explanation is that large, temporary downturns can themselves permanently damage an economy's productive capacity. This hysteresis view, according to which changes in current aggregate demand can have a significant effect on future aggregate supply, dates back to the 1980s but recently underwent a surge of interest in the wake of the Great Recession (e.g., Yellen, 2016). While the two sets of explanations may be observationally similar, they have very different normative implications. If exogenous structural factors drive slow growth, countercyclical policy may be unable to resist or reverse this trend. In contrast, if temporary downturns themselves lead to persistently or permanently slower growth, then countercyclical policy, by limiting the severity of downturns, may have a role to play to avert such adverse developments.[2]

In this paper, we present a theory in which hysteresis might occur and countercyclical monetary policy can moderate its impact if timed appropriately. In our model, hysteresis can arise because workers lose human capital whilst unemployed and unskilled workers are costly to retrain, as in Pissarides (1992). In the presence of nominal rigidities and a zero lower bound (ZLB) constraint on monetary policy, large adverse fundamental shocks can cause recessions whose legacy is persistent or permanent unemployment. Under this setting, the timing of monetary policy matters significantly for long-term outcomes. Timely commitment to future accommodative policy early in a recession can prevent hysteresis from taking root and enable a swift recovery. In contrast, delayed monetary policy interventions may be powerless to bring the economy back to full employment.

Our environment is an economy with downwardly sticky nominal wages and labour search frictions. Human capital depreciates during unemployment spells and unskilled workers are costly to retrain. These features generate multiple steady states. One steady state is a high pressure economy: job finding rates are high, unemployment is low and job-seekers are highly skilled. While tight labour markets - by improving workers' outside options - cause wages to be high, firms still find job creation attractive, as higher wages are offset by low average training costs when job-seekers are mostly highly skilled. The economy, however, can also be trapped in a low pressure steady state. In this steady state, job finding rates are low, unemployment is high, and many job-seekers are unskilled as long unemployment spells have eroded their human capital. Slack labour markets lower the outside options of workers and drive wages down, but hiring is still limited as firms find it costly to retrain these workers. Crucially, this is an *unemployment trap* - an economy near the low pressure steady state can

[1] Here by *secular stagnation* we refer to the literature arguing that a chronic excess of global savings relative to investment has depressed equilibrium real interest rates. This imbalance has been attributed to permanent changes in either borrowing constraints, supply of safe assets, demographics, inequality or monopoly power. See, for example, Eggertsson and Mehrotra (2014) and Caballero and Farhi (2017), among many others.

[2] In the words of Yellen (2016): *"...hysteresis would seem to make it even more important for policymakers to act quickly and aggressively in response to a recession, because doing so would help to reduce the depth and persistence of the downturn, thereby limiting the supply-side damage that might otherwise ensue."*

never self-correct and return to a high pressure state.

Nominal wage rigidities and a lower bound constraint on monetary policy can enable temporary shocks to permanently move the economy from a high pressure steady state into an unemployment trap. Consider an environment where the monetary authority pursues what we term *conventional* monetary policy (CMP): it aims to implement the highest employment level consistent alongside zero nominal wage inflation. A large but temporary increase in households' discount factor raises desired savings, pushing the real interest rate below zero. Monetary policy tries to accommodate the excess demand for savings by lowering the nominal interest rate below zero, but is constrained by the ZLB. Consequently, current prices are forced to adjust downwards, as households' demand for current consumption relative to the future declines. Under downward nominal wage rigidity, the decline in prices causes real wages to rise, and hiring to fall, lengthening average unemployment duration and increasing the incidence of skill loss. Deterioration in average skill quality among the unemployed in turn raises the effective cost of job creation, discouraging vacancy posting and slowing the economy's return to the high pressure steady state even after the shock has abated. In the event of a large enough shock, the economy may be pushed into an unemployment trap from which it is powerless to escape.

The transition to an unemployment trap following a large adverse shock can be avoided if, rather than pursuing CMP, the monetary authority instead engages in what we term *unconventional* monetary policy (UMP). By instead committing to temporarily higher inflation after the liquidity trap has ended, the monetary authority can mitigate both the initial rise in unemployment, and its persistent (or permanent) negative consequences. UMP, however, is only effective if it is implemented early in the downturn, before the recession has left substantial scars. Once the skill composition of the unemployed has significantly worsened following the shock, monetary policy cannot undo the high cost of hiring through the promise of higher future prices. With nominal wages free to adjust upwards, any attempt to generate price inflation is met by nominal wage inflation, leaving real wages unaffected. Thus, once hysteresis has taken root, monetary policy cannot undo it. In such cases, fiscal policy, in the form of hiring or training subsidies, is necessary to engineer a recovery.

One might think that it is more natural to address hysteresis through such fiscal policies more generally. In our model, as in the standard New Keynesian (NK) model, an appropriately rich set of fiscal instruments would wholly obviate the need for countercyclical monetary policy (Correia et al., 2008). But in reality, fiscal policy can be imperfect and slow to respond to a downturn, leaving monetary policy as the first responder when it comes to countercyclical stabilisation. The NK literature on stabilisation policy concentrates on monetary rather than fiscal policy for precisely this reason. Given the limitations of fiscal policy, it is important to know if and when monetary policy can prevent hysteresis or mitigate its effects. Moreover, even if fiscal policies such as training subsidies were used to engineer a recovery, such policies may not be costless as households must forgo consumption when more resources are allocated towards training unskilled workers. Unconventional monetary policy – which prevents any initial rise in unemployment – avoids these costs altogether.

Our paper underscores the importance of *timely* monetary policy accommodation. In the presence of hysteresis, a failure to deliver stimulus early on in a recession can have irreversible costs. This contrasts with standard NK models, in which accommodative policies are equally effective at any point in a liquidity trap (Eggertsson and Woodford, 2003). In fact, these models predict that while overly tight policy may be costly in the short-run, it has no long-run consequences, since temporary shocks have no permanent effects in stationary models. Consequently, timeliness of monetary accommodation is not particularly relevant in these models, as delaying accommodation does not reduce welfare in the long run. Our model instead focuses on a monetary economy with multiple steady states. Thus, monetary policy can affect not just fluctuations around a steady state, but also the level of steady state activity.[3]

Our focus on multiple steady states also distinguishes our paper from recent work on the persistent effects of recessions (Benigno and Fornaro, 2017; Schmitt-Grohe and Uribe, 2017). These papers study economies which can switch to a bad equilibrium featuring permanently low or negative inflation, a binding ZLB, and high unemployment. This bad equilibrium is the result of self-fulfilling pessimistic beliefs; equally, self-fulfilling optimism can return the economy to the good equilibrium. Our analysis differs in two ways. First, in our model, high unemployment can persist even after monetary policy is no longer constrained by the ZLB. Second, it features path dependence: optimistic beliefs cannot return the economy back to full employment if the economy is stuck in an unemployment trap, nor can they speed up a slow recovery.

This is because dynamics in our economy are driven by a slow-moving state variable - the fraction of unskilled job-seekers. Even if a swift recovery is anticipated, this does not induce firms to hire and train relatively unskilled job-seekers today. In fact, firms postpone hiring, preferring to wait until there are more skilled job-seekers. Since hiring falls, the skill composition of job-seekers actually worsens and the firms' optimism is self-defeating. Since self-fulfilling optimism cannot escape the trap ex post, it is all the more important to avoid it ex ante.

The remainder of the paper is structured as follows. Next we discuss related literature. Section 2 presents the model economy. Section 3 characterises steady states and equilibrium under flexible wages. Section 4 introduces nominal rigidities, and studies how demand shocks can cause slow recoveries or permanent stagnation. Section 5 presents some extensions and discussions. Section 6 concludes.

Related literature Our paper relates most closely to a small number of recent studies analysing hysteresis and monetary policy in the presence of nominal frictions. Benigno and Fornaro (2017) present a model in which pessimism can drive the economy to the ZLB, reducing firms' incentive to innovate and giving rise to persistent or permanent slowdowns. A commitment to alternative monetary policy rules (or subsidies to innovation) can help avoid or exit such *stagnation traps*. While we study a different channel through which hysteresis might operate, and focus on unemployment rather than output hysteresis, our results resonate with theirs. A key difference is that in our model, a commitment to accommodative monetary policy can only avoid an unemployment trap if it is implemented swiftly; if the economy is already stuck in such a trap, monetary policy is of little help. Bianchi et al. (2019) also find that declines in R&D during recessions can explain persistent effects of cyclical shocks on growth, while Garga and Singh (2021) study the conduct of optimal monetary policy in a model

[3] This does not mean that monetary policy can manipulate a long-run trade-off between inflation and unemployment. In our baseline model, once the economy has converged to a particular steady state unemployment rate, monetary policy is powerless to reduce unemployment below this rate.

embedding this feature. Laureys (2014) studies monetary policy in an environment, similar to ours, where skills depreciate during unemployment spells, but focuses on linear dynamics around a unique steady state. In the same vein, Galí (forthcoming) studies optimal monetary policy in a NK model where insider-outsider labour markets can generate hysteresis. In these two papers, however, temporary shocks and policy mistakes have persistent but not irreversible effects. By studying an environment where temporary shocks can cause irreversible damage, we are able to stress the distinctive role of timeliness in monetary policy action.

The literature studying models with multiple steady-states has explored the critical role of *timely* policy interventions in these environments. However, this literature has largely abstracted from nominal rigidities and monetary policy. Drazen (1985) argues that the loss of human capital due to job loss in recessions can lead to delayed recoveries. den Haan (2007) uses a labour search model with multiple steady-states to argue that one-time shocks may lead to permanently higher unemployment when unemployment benefits are generous. Schaal and Taschereau-Dumouchel (2016) show that a labour search model with aggregate demand externalities can generate additional persistence in labour market variables. Similarly, in Schaal and Taschereau-Dumouchel (2015), large recessions frustrate coordination on a high-activity equilibrium, allowing temporary shocks to cause quasi-permanent recessions. Our model instead draws on Pissarides (1992), who argued that skill depreciation can give rise to multiple steady states. Sterk (2016) studies a quantitative version of Pissarides (1992)'s model and argues that it can account for the behaviour of job finding rates in the United States. Relative to our work, all these studies consider purely real models. As such, they do not address the question we are interested in - namely, how monetary policy should be conducted in the presence of hysteresis.

On the empirical side, a large literature finds evidence of drops in productive capacity after recessions. Dickens (1982) finds that recessions can permanently lower productivity; Haltmaier (2012) finds that trend output falls by 3 percentage points on average in developed economies four years after a pre-recession peak. Using cross-country data, Martin et al. (2014) find that severe recessions have a sustained and sizeable negative impact on output. Similarly, Ball (2014) finds that countries with a larger fall in output during the Great Recession experienced a larger decline in *potential output*. Within the U.S., Yagan (2019) finds that states exposed to larger unemployment shocks in 2007 experienced significantly lower employment rates in 2015. Song and von Wachter (2014) find that the persistent decline in employment following job displacement is larger during recessions, suggesting that a spike in job destruction rates can persistently affect unemployment.

Our work also contributes to the large literature exploring how downward nominal wage rigidity can exacerbate unemployment outcomes during a recession. While the literature has largely examined how downward nominal wage rigidities can raise the spectre of layoffs during recessions (e.g. Murray 2019), our paper highlights how such rigidities can discourage job creation by raising the effective cost of hiring. A large and growing empirical literature supports our assumption that nominal wages are downwardly rigid. Using payroll data, Grigsby et al. (2021) find that only 2.4% of all workers observe a nominal wage cut during a year and that wages of new hires do not appear to be more flexible than those of incumbent workers. Further, at the height of the Great Recession, they find that only 6% of workers observed a nominal wage cut. Barattieri et al. (2014) and Fallick et al. (2020) find similar evidence that nominal wage cuts are infrequent using data from the Survey of Income and Program Participation (SIPP) and the Bureau of Labor Statistics (BLS), respectively. Finally, using data on the wages of new hires at the *job level*, Hazell and Taska (2019) find that nominal wage cuts account for only 9% of the adjustments in new hires' posted wages.

Beyond the literature on hysteresis and downward nominal wage rigidities, our analysis also connects to a few recent developments in monetary economics. Like us, Dupraz et al. (2019) study a *plucking model* in which downward nominal wage rigidity gives rise to asymmetric effects of monetary policy: while deflation can lead to an increase in real wages and a fall in hiring, inflation has limited effects on unemployment. In their model, this asymmetry increases the costs of business cycles despite shocks having at most a temporary effect. Our analysis suggests that the costs associated with this asymmetry become amplified when combined with hysteresis: temporary deflation can lead to permanently higher unemployment and deterioration in the skill composition of the unemployed, both of which cannot be reversed by higher inflation in the future. This underscores the relevance of timely monetary policy to stabilise employment, even at the cost of compromising price stability. Our result resonates with Berger et al. (2016) and Acharya et al. (2020), who find that monetary policy should prioritise output and employment stabilisation over price stability when households are imperfectly insured. Our analysis provides another reason why employment fluctuations might have higher costs, and warrant more attention.

Finally, our paper relates to the secular stagnation literature. Eggertsson and Mehrotra (2014) and Caballero and Farhi (2017) present models in which the market clearing interest rate is persistently or permanently negative, leading to persistently low output, as the ZLB prevents nominal rates from falling to clear markets. In such situations, a permanent change in fiscal or monetary policy is typically required to prevent stagnation. We share this literature's concern with long run outcomes, but consider a different mechanism: in our model *temporary* falls in market clearing interest rates have permanent effects, which *temporary* monetary accommodation can prevent.

2 The Model Economy

We start by presenting a benchmark economy with labour market frictions à la Diamond-Mortensen-Pissarides (DMP) and no nominal rigidities. Time is discrete and there is no uncertainty. The only addition to the standard DMP model is that we assume that workers can lose skill following an unemployment spell.

Workers There is a unit mass of risk-neutral ex ante identical workers with discount factor β. Workers trade nominal bonds which pay a nominal return of $1 + i_t$. Workers can either be employed or unemployed. We denote the mass of employed workers as n and unemployed as $u = 1 - n$. Unemployed workers produce $b > 0$ as home production. The stock of employed workers evolves as:

$$n_t = [1 - \delta(1 - q_t)]\, n_{t-1} + q_t u_{t-1}, \tag{1}$$

where δ is the exogenous rate at which workers are separated from their current jobs and q_t is the job finding rate. (1) implies that a worker separated at the beginning of period t can find another job within the same period. Next, let $\mathbb{W}_t$ denote the value of an employed worker and $\mathbb{U}_t$ denote the value of an unemployed worker at time t. These can be expressed as follows:

$$\mathbb{W}_t = \omega_t + \beta\Big\{ [1-\delta(1-q_{t+1})]\,\mathbb{W}_{t+1} + \delta(1-q_{t+1})\mathbb{U}_{t+1}\Big\}, \tag{2}$$

$$\mathbb{U}_t = b + \beta\Big\{q_{t+1}\mathbb{W}_{t+1} + (1-q_{t+1})\mathbb{U}_{t+1}\Big\}, \tag{3}$$

where ω_t denotes the real wage at date t.

labour market In the spirit of Pissarides (1992), we assume that a worker who gets separated from her job and who fails to transition back to employment by the end of a period immediately loses the skills that she acquired while employed. That is, any worker unemployed for at least 1 period becomes *unskilled*. Because unskilled workers produce zero output when matched with a firm, a firm that hires an unskilled worker must pay a training cost $\chi > 0$ to use that worker in production. Once the firm trains the worker, she remains *skilled* until the next unemployment spell of at least 1 period. Let μ_t denote the fraction of unskilled workers in the pool of job-seekers (l_t) at date t. This fraction is defined as:

$$\mu_t = \frac{u_{t-1}}{l_t} \equiv \frac{u_{t-1}}{1-(1-\delta)(1-u_{t-1})}. \tag{4}$$

(4) shows that a higher level of unemployment in the past corresponds to a higher fraction of unskilled job-seekers. As such, there is a one-to-one mapping between u_{t-1} and μ_t.

Matching technology Search is random. The number of successful matches m_t between job-seekers l_t and vacancies v_t is given by a CRS matching technology $m(v_t, l_t)$. We define market tightness θ_t as the ratio of vacancies to job-seekers. The job finding probability of a job-seeker, q_t, and the job filling probability of a vacancy, f_t, are then given by:

$$q(\theta_t) = \frac{m(v_t, l_t)}{l_t} \qquad \text{and} \qquad f(\theta_t) = \frac{m(v_t, l_t)}{v_t} = \frac{q(\theta_t)}{\theta_t}. \tag{5}$$

Firms A representative CRS firm uses labour as an input to produce the final good. The production function is given by $y_t = An_t$ where $A > b$ is aggregate productivity and n_t is the number of employed workers in period t. A firm must incur a vacancy posting cost of $\kappa > 0$ and an additional training cost of χ for each unskilled worker hired. A firm with n_{t-1} workers at the beginning of period t chooses vacancies (taking wages as given) to maximise lifetime discounted profit:

$$\mathbb{J}_t = \max_{v_t \geq 0} (A-\omega_t)n_t - (\kappa + \chi\mu_t f_t)v_t + \beta\mathbb{J}_{t+1} \qquad \text{s.t.} \qquad n_t = (1-\delta)n_{t-1} + f_t v_t,$$

where ω_t is the wage paid to all workers. Importantly, the total cost of job creation depends on the skill composition of job-seekers. Since the firm pays a cost χ to train each unskilled job-seeker it hires, the effective average cost of creating a job is increasing in the fraction of unskilled job-seekers μ_t. From (4), μ_t depends on past unemployment rates, making the cost of job creation *increasing* in the unemployment rate. The value of a filled vacancy, $J_t = \partial \mathbb{J}_t / \partial n_t$, can be written as

$$J_t = A - \omega_t + \beta(1-\delta)J_{t+1}. \tag{6}$$

Free entry of vacancies implies

$$J_t \leq \frac{\kappa}{f_t} + \chi\mu_t \ , \ \theta_t \geq 0 \ , \ \textit{with at least one strict equality.} \tag{7}$$

Resource constraint The resource constraint can be written as

$$c_t = An_t + b(1-n_t) - (\kappa + \chi\mu_t f_t)\, v_t.$$

Wage and price determination While we ultimately seek to analyse the conduct of monetary policy in an environment with nominal wage rigidities, it is useful to first study a flexible wage benchmark, in which wages are simply determined by Nash bargaining every period. Because bargaining occurs after all hiring and training costs have been paid, all workers are paid the same wage.[4] Formally, the Nash bargaining problem is $\max_{\omega_t} J_t^{1-\eta} (\mathbb{W}_t - \mathbb{U}_t)^\eta$ where $\eta \in [0,1)$ denotes the bargaining power of the workers. The Nash-bargained wage is

$$\omega_t^* \quad = \quad \eta A + (1-\eta)b + \beta(1-\delta)\eta q_{t+1} J_{t+1}. \tag{8}$$

Crucially, an increase in next period's job finding rate puts upward pressure on the wage because it increases the worker's outside option. Substituting (8) into (6) yields

$$J_t = a + \beta(1-\delta)(1-\eta q_{t+1})J_{t+1}, \tag{9}$$

where $a \equiv (1-\eta)(A-b)$. (9) implies that the value of a filled vacancy to a firm lies in the interval: $J_t \in [J_{min}, J_{max}]$, for $J_{min} \equiv a/[1-\beta(1-\delta)(1-\eta)]$ and $J_{max} \equiv a/\left[1-\beta(1-\delta)\right]$.[5] It also implies that an increase in the job finding rate at every future date, through an upward pressure on the wage, results in a smaller profit to the firm and thus a lower J_t. In this flexible wage benchmark, the classical dichotomy holds and the price level does not affect real allocations. Thus, it is not necessary to describe the conduct of monetary policy. Equilibrium dynamics in the benchmark economy is completely characterised by (7), (9) and the law of motion for μ_t, which is given by:

$$\mu_{t+1} \quad = \quad \frac{1-q(\theta_t)}{1+(1-\delta)[1-q(\theta_t)-\mu_t]}. \tag{10}$$

[4]Both skilled and unskilled workers have the same outside option since training costs are sunk at the time of bargaining and all job-seekers have the same probability of finding a job.

[5]J_{min} is the lowest value of a filled vacancy and is achieved when firms expect $q_t = 1$ forever. Conversely, J_{max} is the value of a filled vacancy when firms expect $q_t = 0$ forever (labour markets are expected to be the slackest forever).

For analytical tractability, in Sections 3 and 4, we assume a particular form for the matching function, $m_t = \min\{v_t, l_t\}$, which implies $q(\theta_t) = \min\{\theta_t, 1\}$, $f(\theta_t) = \min\{1, 1/\theta_t\}$. In particular, it implies that the short side of the market matches with probability 1. We refer to the case with $\theta_t < 1$ as the *slack labour market regime* and the one with $\theta_t \geq 1$ as the *tight labour market regime.* In Online Appendix L, we explore the quantitative implications of our model when we use a more standard matching function, such as the CES matching function.

3 Flexible wage benchmark

Our goal is to study how temporary shocks can scar the economy permanently depending on the conduct of monetary policy. Permanent scarring is possible in our model because it features multiple steady states, and shocks can push the economy from one steady state to another. In this section, we explain why multiple steady states exist in our economy and characterise dynamics in the flexible wage benchmark.

3.1 Steady states

In our model, multiplicity of steady state unemployment rates arises naturally because workers lose skill while unemployed and firms must pay a cost to train unskilled workers. Consider an economy with high unemployment. Since average unemployment duration is high, the fraction of unskilled job-seekers is high. Consequently, firms must spend more on training workers, which discourages them from creating vacancies, even though slack labour markets lower workers' outside options and drive down wages. Thus, a high unemployment rate is self-sustaining. Conversely, when unemployment is low, mean unemployment duration is low and the fraction of skilled job-seekers is high. While wages are high since tight labour markets improve workers' outside options, firms still post vacancies because expected training costs are low as most job-seekers are skilled. This in turn sustains low unemployment. Given our Leontief matching function, the low unemployment steady state corresponds to zero unemployment. Such a steady state exists under the following assumption.

Assumption 1. *Vacancy posting costs are low enough: $\kappa < J_{min}$.*

The law of motion for employment (1) implies that full employment ($n = 1$) requires $q = 1$ (and $f = 1/\theta \leq 1$); job-seekers are on the short side of the market, and always find a job within one period. Skill depreciation never occurs, and the law of motion for the skill composition (10) implies $\mu = 0$. Thus, the effective cost of hiring a worker is simply κ/f and the job creation condition (7) becomes $J_{min} = \kappa\theta^{\text{fe}}$ in steady state, where θ^{fe} denotes the labour market tightness associated with full employment. Assumption 1 ensures that this equation has a solution featuring $\theta^{\text{fe}} > 1$.

While skill depreciation *can* generate multiple steady states, whether it in fact does so depends on the strength of the scarring effects of unemployment (measured by χ) and the sensitivity of wages to workers' outside options (measured by η). The following assumption ensures that both forces are strong enough such that in addition to the full employment steady state, there exist additional interior steady states featuring higher unemployment.

Assumption 2. *The training cost χ is neither too small nor too large, i.e., $\chi \in (\underline{\chi}, J_{max} - \kappa)$, and the workers' bargaining power is not too small, $\eta > \underline{\eta}$. The thresholds $\underline{\eta}$ and $\underline{\chi}$ are defined in Appendix B.*

Appendix B shows that $\kappa + \chi < J_{max}$ ensures that training costs are not too large, so that the worst steady state features a positive level of employment.[6] The remaining elements of Assumption 2 ensure that two interior steady states with unemployment exist (in addition to the full employment steady state). From the law of motion for employment (1), at any interior steady state ($n < 1$), firms are on the short side of the labour market ($q < 1$). This implies that there is some skill depreciation ($\mu > 0$), since from the law of motion for the skill composition (10), we have $q = 1 - \mu < 1$ in steady state. At an interior steady state, the job creation condition (7) becomes[7]

$$\frac{a}{1 - \beta(1-\delta)\left[1 - \eta(1-\mu)\right]} = \kappa + \chi\mu. \tag{11}$$

The left-hand side (LHS) of (11) is the value of a filled vacancy with $q = 1 - \mu$, while its right-hand side (RHS) is the cost of creating a job. (11) describes a quadratic equation in μ, which has at most two solutions. Appendix B shows that Assumption 2 guarantees that economically meaningful solutions to this equation exist. A high bargaining power η increases the sensitivity of wages and profits to labour market conditions. When unemployment is low, wages are high because the worker's outside option is relatively favourable. Firms are willing to tolerate high wages because training costs are low. When labour markets are slack and unemployment is high, workers are relatively unskilled and expensive to train; firms are willing to pay the high training costs because wages are relatively low.

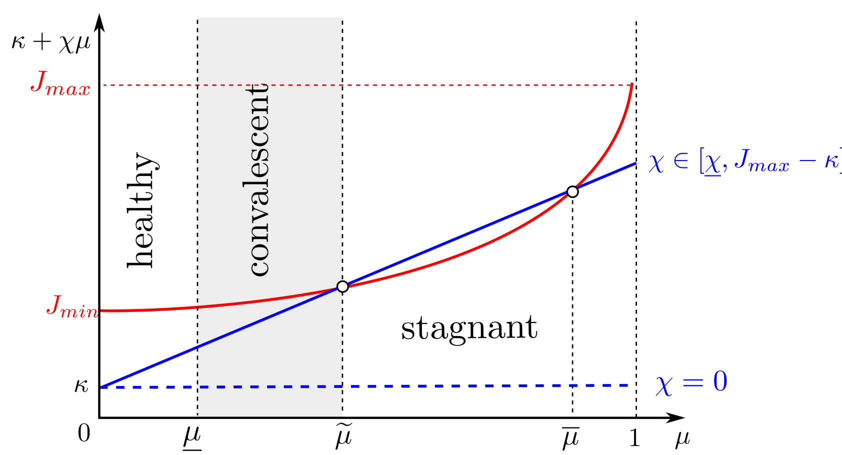


Figure 1. Multiple steady states: The convex curve depicts the LHS of (11), the straight line depicts the RHS.

Figure 1 graphically depicts the arguments above. The convex curve plots the LHS of (11), while the straight lines plot its RHS for different values of χ. When χ is too low, the two curves do not

[6] If there was zero employment in steady state, everyone would be unskilled ($\mu = 1$) and labour markets would be completely slack ($q = 0$). Since wages are the lowest they can be, the value of a filled vacancy is J_{max}, the highest it can be. Since all workers are unskilled, the effective cost of hiring a worker is now $\kappa + \chi$. $\kappa + \chi < J_{max}$ ensures that at even such a high level of μ, a firm would find it profitable to post some vacancies, ruling out the uninteresting possibility of a zero employment steady state. Qualitatively, none of our results would change if we allowed for a zero employment steady state.

[7] In this section and in what follows, it will be convenient to work with the fraction of unskilled workers μ rather than the unemployment rate u as the state variable of interest. Equation (4) defines a one-to-one map between μ_t and u_{t-1}.

intersect and there are no interior steady states. If χ is too high, the straight line lies above the convex curve at $\mu = 1$ and there exists a zero-employment steady state, in violation of Assumption 2. When χ is in the appropriate range, then there are two interior steady states, $\widetilde{\mu}$ and $\overline{\mu}$ (with $\widetilde{\mu} < \overline{\mu}$). Finally, recall that there is always a full employment steady state at $\mu = 0$. The three steady states are associated with different degrees of market tightness, and accordingly, with different levels of wages: wages at the full employment steady state are higher than at the moderate unemployment steady state $\widetilde{\mu}$, which in turn are higher than at the high unemployment steady state $\overline{\mu}$.

Multiple steady states under alternative search specifications Our model is highly stylised. In reality, firms may be able to distinguish between skilled and unskilled applicants – either when interviewing them or even before, when they specify the job-requirements when posting a vacancy. In Online Appendix K, we extend the model in various dimensions and show that skill depreciation generally results in the existence of multiple steady states, even when firms can test workers' skills, or post vacancies in segmented markets.

3.2 Dynamics

Next, we characterise the transitional dynamics of the economy starting from any $\mu_0 \in [0, 1]$. While we have thus far not introduced any aggregate shocks which would move the economy away from a steady state, we will do so in Section 4. For now, we can think of the experiment as studying the evolution of the economy after past shocks have moved it to a point μ_0. The subsequent evolution of the economy can be described by a mapping $\mu_{t+1} = \mathcal{M}(\mu_t)$.

As indicated in Figure 1, the state space can be partitioned into 3 regions, depending on the initial fraction of unskilled job-seekers μ: (i) a *healthy region* featuring low unemployment and a highly skilled workforce (low μ), (ii) a *convalescent region* featuring moderate levels of unemployment and a moderately skilled workforce (intermediate level of μ); and finally (iii) a *stagnant region* with high unemployment and a largely unskilled workforce (high μ). Dynamics differ between these three regions, as we now describe.

Healthy region If the economy starts in the healthy region, defined as $\mu \in [0, \underline{\mu}]$ where $\underline{\mu} \equiv (J_{min} - \kappa)/\chi < \widetilde{\mu}$, then labour markets are tight and the economy immediately converges back to the full employment steady state, as formalised in the following proposition.

Proposition 1. *Suppose the economy starts in the healthy region, $\mu_0 < \underline{\mu}$. Then the economy converges to the full employment steady state in one period, $\theta_t = (J_{min} - \chi\mu_0)/\kappa$ for $t = 0$, and $\theta_t = J_{min}/\kappa > 1$, $n_t = 1$, $\mu_t = 0$ for $t \geq 1$. Furthermore, the value of a filled vacancy and wages are always at their full employment steady state level: $J_t = J_{min}$ and $\omega_t = \omega^*_{\text{fe}} \equiv \eta A + (1 - \eta)b + \beta\eta(1 - \delta)J_{min}$ for all $t \geq 0$.*[8]

Proof. See Appendix C. □

[8] The equilibrium is unique, except in the knife-edge case where $\mu = \underline{\mu}$, where there also exists other equilibria in which the economy returns to the full employment steady state in 2 periods instead of 1. Note, however, that in these equilibria the value of a filled vacancy and the real wage also satisfy $J_t = J_{min}$, $\omega_t = \omega^*_{\text{fe}}$ for all $t \geq 0$.

Intuitively, when the unemployment rate is very low, average skill quality of job-seekers is very high. Hence, low training costs make it attractive for firms to post enough vacancies to absorb all job-seekers, despite the high wages associated with tight labour markets in the present and future. Consequently, unemployment duration is short (everyone finds a job by the end of the first period), and the skill quality of the workforce remains high. While we have not yet introduced any shocks, one interpretation is that the full employment steady state is stable with respect to shocks which only cause small deterioration in the average skill composition of job-seekers. In particular, if μ_0 rises to a level in the interval $(0, \underline{\mu}]$, the effect of the shock is immediately reversed as job-seekers are still largely skilled, and firms are willing to post enough vacancies to hire and retrain all job-seekers on the spot. As long as $\mu_0 < \underline{\mu}$, $\theta_0 > 1$ and the economy immediately returns to full employment: $\mu_1 = \mathcal{M}(\mu_0) = 0$.

Convalescent region If the economy starts in the convalescent region, $\mu_0 \in (\underline{\mu}, \widetilde{\mu})$, it eventually returns to full employment, but does not do so instantaneously:

Proposition 2 (Dynamics in the convalescent region)**.** *For β sufficiently close to 1, there exists a unique strictly increasing sequence $\{\mu^n\}_{n=0}^{\infty}$ with $\mu^0 \equiv \underline{\mu}$ and $\lim_{n\to\infty} \mu^n = \widetilde{\mu}$, such that if $\mu_0 \in I^n \equiv (\mu^{n-1}, \mu^n]$, the economy reaches the healthy region in n periods and reaches the full-employment steady-state in $n+2$ periods, i.e., $\mu_n = \underline{\mu}$, $\mu_{n+1} \in (0, \underline{\mu})$ and $\mu_{n+2} = 0$. Furthermore:*

1. *Recoveries can be arbitrarily long: As $\mu_0 \to \widetilde{\mu}$, the time it takes for the economy to return to the healthy region tends to infinity.*[9]

2. *Recoveries can be arbitrarily slow: If μ_0 is close to $\widetilde{\mu}$, then μ declines very slowly early on in the recovery.*[10]

Proof. See Appendix D. □

Figure 2 illustrates the gradual decline in μ_t, described in Proposition 2, by depicting the equilibrium starting from a point μ_0 in the convalescent region. The horizontal axis denotes μ_t, the vertical axis denotes μ_{t+1}, and the kinked curve denotes the function $\mu_{t+1} = \mathcal{M}(\mu_t)$. In the example depicted, μ_0 is shown to lie in the interval $I^5 = (\mu^4, \mu^5]$, so it takes 5 periods for the economy to reach the healthy region, and 7 periods to reach full employment. During the transition, employment is growing over time and the fraction of unskilled job-seekers is shrinking. As long as the economy is in the convalescent region, labour markets are slack and real wages are low, $\omega_t^* < \omega_{\text{fe}}^*$. But as soon as the economy reaches the interior of the healthy region, labour markets become tight and real wages reach their steady state level ω_{fe}^*. The real wage level $\omega^*(\mu_t)$, which we will refer to as the *natural real wage*, will play an important role in our analysis of monetary policy in Section 4.1.[11]

[9] Formally, for any $T \in \mathbb{N}$, there exists $\varepsilon > 0$ such that if $\mu_0 \in (\widetilde{\mu} - \varepsilon, \widetilde{\mu})$, $\mu_t > 0$ for all $t < T$.

[10] Formally, for any $\delta > 0$, $T \in \mathbb{N}$, there exists $\varepsilon > 0$ such that if $\mu_0 \in (\widetilde{\mu} - \varepsilon, \widetilde{\mu})$, $\mu_t > \mu_0 - \delta$ for all $t < T$.

[11] Sterk (2016) also presents a search model which features multiple steady states and slow recoveries. However, the mechanisms which drive such outcomes in his model are different than in our model. Sterk (2016) assumes that the workers have bargaining power $\eta = 0$, implying that wages are always equal to b regardless of the fraction of unskilled workers. Thus, the *natural real wage* in his model is constant. In contrast, in our model, a time-varying natural wage is an important aspect of a slow recovery, as explained above.

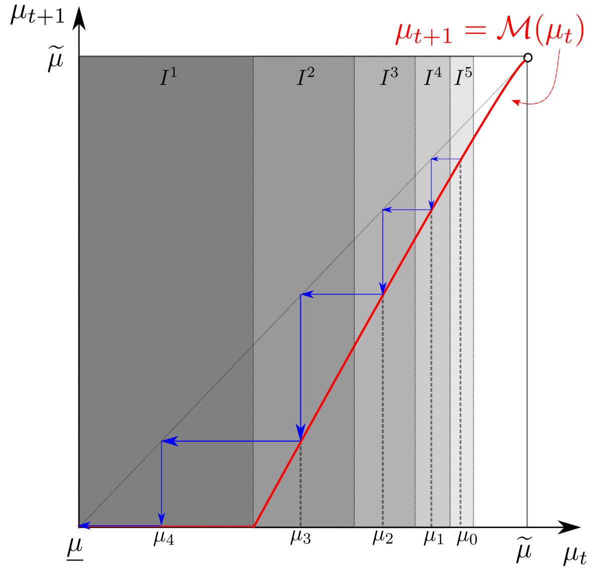


Figure 2. Dynamics in convalescent region: the kinked curve denotes the function $\mu_{t+1} = \mathcal{M}(\mu_t)$, the arrows depict the equilibrium trajectory of μ_t.

When the fraction of unskilled job-seekers μ_0 is higher than $\underline{\mu}$, expected training costs $\chi\mu_0$ are so high that firms are unable to recoup these costs if wages are high and expected to stay so. Slack labour markets must therefore persist for some time for wages to be low and for firms to still be willing to post vacancies at date 0. In other words, in equilibrium, the labour market must experience a *slow recovery*. In fact, the speed of the recovery decreases in the initial fraction of unskilled job-seekers. A higher μ_0 requires a lower job finding rate for wages to be driven down and firms to be induced to post vacancies. But such a low job finding rate in turn reduces the rate at which unskilled job-seekers are re-hired and regain skill. In the convalescent region, $\mu_t - \mu_{t+1}$ (the gap between the 45 degree line and the kinked line in Figure 2) is a decreasing function of μ_t: the worse the current state of the labour market, the slower it recovers. Accordingly, the economy can spend an arbitrary long time in the convalescent region before transitioning to the healthy region. When the economy starts deep in the convalescent region (μ_0 close to $\widetilde{\mu}$) the recovery takes disproportionately longer (point 1 of Proposition 2), and the fraction of unskilled job-seekers declines at a slower rate in the early stage of the recovery (point 2).

Stagnant region If the economy starts in the stagnant region, $\mu_0 \in [\widetilde{\mu}, 1]$, it never returns to full employment. When $\mu_0 \in [\widetilde{\mu}, 1]$, expected training costs are so high that they discourage firms from posting enough vacancies to bring the economy out of this region. Importantly, this is *not* because real wages are sticky. In the stagnant region, the high fraction of unskilled job-seekers, μ_t, is accompanied by low real wages which induce firms to post some vacancies. But such low real wages can only be sustained by low job finding rates, which in turn prevent unskilled workers from being hired and retrained in sufficient numbers for the economy to escape the stagnant region. This region is an *unemployment trap*, as the following proposition states.

Proposition 3 (Unemployment traps)**.** *If the economy is pushed into the stagnant region, i.e.,* $\mu_t \geq \widetilde{\mu}$, *then it never returns to the full employment steady state.*

Proof. See Appendix E. □

The area to the right of $\widetilde{\mu}$ in Figure 1 depicts the stagnant region. Starting from any μ_0, there is an equilibrium in which μ_t converges to the high unemployment steady state $\overline{\mu}$.[12] Thus, if the economy starts in the stagnant region, it never escapes.

4 Nominal rigidities

The analysis above highlighted that starting from a high level of unemployment, the economy may be unable to return to full employment. With nominal rigidities, this means that if monetary policy fails to act quickly and prevent large adverse shocks from significantly increasing unemployment, such shocks can have persistent or permanent consequences.

Shocks We focus on the economy's response to a temporary demand shock, modelled as a temporary increase in households' patience: $\beta_0 > 1$, $\beta_t = \beta < 1$ for all $t > 0$. The NK literature has used this type of shock to capture an increase in the supply of savings which pushes the real interest rate below zero.[13] We prefer to focus on a temporary demand shock (rather than, e.g., a productivity shock) since such a shock can only have persistent effects in the presence of nominal rigidities.

Nominal rigidities The model specified in the previous section is characterised by the classical dichotomy and thus, monetary policy is unable to affect allocations. Since our objective is to understand whether monetary policy can prevent or moderate hysteresis, we break the classical dichotomy by introducing nominal rigidities in the form of downwardly sticky nominal wages. In particular, we suppose that at any date t the nominal wage must satisfy $W_t \geq \varphi W_{t-1}$ where $\varphi \in (0, 1]$ limits how much nominal wages can fall between dates $t-1$ and t ($\varphi = 1$ means that nominal wages cannot fall, while $\varphi < 1$ implies that nominal wages can adjust downwards to some extent).

In the spirit of Schmitt-Grohe and Uribe (2013), given the current state μ_t, we assume that nominal wages are set to $W_t = \omega^*(\mu_t)P_t$ whenever possible, where $\omega^*(\mu_t)$ is the real wage in the flexible wage benchmark. However, if $\omega^*(\mu_t)P_t < \varphi W_{t-1}$, then $W_t = \varphi W_{t-1}$. That is, the nominal wage is set by Nash bargaining whenever the downward nominal wage rigidity (DNWR) constraint is not violated:

$$W_t = \max\Big\{\varphi W_{t-1}, P_t\omega^*(\mu_t)\Big\}. \tag{12}$$

[12] Appendix E proves that the economy never leaves the stagnant region. To show that the equilibrium which converges to $\overline{\mu}$ is locally determinate, we use numerical methods. We check the stability properties of the high-unemployment steady state by drawing a random sample of a 100,000 parameter combinations, dropping any draw which violates Assumptions 1 and 2. Given a parameter vector, we then linearize our two dynamic equations around the high-unemployment steady state and compute the eigenvalues of this system. Since we have one pre-determined variable (μ_t) and one jump-variable (θ_t), a locally determinate equilibrium converging to $\overline{\mu}$ exists if one eigenvalue is inside the unit circle and one is outside. 100 percent of our feasible draws satisfy this property, indicating that equilibrium is locally determinate in the stagnant region.

[13] In a richer model, such a shock could arise from a tightening of borrowing limits or an increase in precautionary savings motives. See, for example, Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017).

Note that even when the DNWR constraint binds and nominal wages are unable to further adjust downwards, *real* wages can still fall if inflation is positive. Indeed, the real wage is given by

$$\frac{W_t}{P_t} = \omega_t = \max\left\{\varphi\frac{P_{t-1}}{P_t}\omega_{t-1}, \omega^*(\mu_t)\right\}. \tag{13}$$

Our choice of DNWR rather than symmetric wage rigidity is motivated by the overwhelming empirical evidence supporting asymmetric wage rigidities, e.g., Grigsby, Hurst and Yildirmaz (2021), Hazell and Taska (2019), Fallick et al. (2020) and other papers cited in our literature review. These papers present evidence that nominal wages are downwardly sticky for all workers, including new hires.[14]

It is important to note that our results do not require nominal wages of new and existing workers to be *very* downwardly inflexible; all we need is that nominal wages are not perfectly flexible. Furthermore, in all the scenarios we consider, the real wage stays within the bargaining set, and so the workers have no incentive to agree to a further nominal wage cut (beyond the fall from W_{t-1} to φW_{t-1}). If instead, shocks were larger than in the scenarios we consider, leading real wages to leave the bargaining set under specification (12), we could replace (12) with $W_t = \min\left[\max\left[\varphi W_{t-1}, P_t\omega^*(\mu_t)\right], P_t\overline{\omega}_t\right]$ where $\overline{\omega}_t = A + \beta(1-\delta) J_{t+1}$ is the highest wage which lies in the bargaining set. This would imply that firms and workers renegotiate whenever nominal rigidities drive the wage outside the bargaining set. Making this assumption would not change any of our results. In this sense, we do not require that workers and firms forgo mutually beneficial wage cuts and the Barro (1977) critique does not apply.

4.1 Response under Conventional Monetary Policy (CMP)

We first examine how the economy would respond to a temporary demand shock under a regime we term *conventional monetary policy* (CMP).

Description of CMP We assume that the monetary authority sets the nominal interest rate i_t subject to the ZLB, i.e., $i_t \geq 0$. Risk neutrality implies that the equilibrium real interest rate equals $r_t = \beta_t^{-1} - 1$. Inflation then follows from the Fisher equation:

$$\frac{P_{t+1}}{P_t} = \beta_t(1+i_t). \tag{14}$$

Since nominal wages are not perfectly flexible downwards, monetary policy can affect real wages and the level of unemployment by influencing the price level. We specify conventional monetary policy as a standard interest rate rule responding, whenever possible, to deviations of the price level from a state-dependent price level target $P_t^\star$ compatible with the real wage being at its natural level for an unchanged nominal wage:

$$1 + i_t = \max\left\{\beta_t^{-1}\left(\frac{P_t}{P_t^\star}\right)^{\phi_\omega}, 1\right\} \qquad \text{where} \qquad P_t^\star = \frac{W_{t-1}}{\omega^*(\mu_t)}. \tag{15}$$

[14] The joint assumption of downwardly rigid wages and fully flexible prices is not critical for our result that a temporary negative discount factor shock raises unemployment. Online Appendix M presents a model with sticky prices and wages, and shows that this result holds regardless of the relative degree of price and wage stickiness.

It is worth noting that implementing the price level $P_t = P_t^\star$ results in the lowest possible unemployment rate given the state of the economy, while keeping nominal wage inflation at zero. To see this, note first that dividing (12) by P_t and W_{t-1} respectively yields:

$$\frac{W_t}{P_t} = \max\left\{\varphi \frac{W_{t-1}}{P_t}, \omega^*(\mu_t)\right\} \qquad \text{and} \qquad \frac{W_t}{W_{t-1}} = \max\left\{\varphi, \frac{P_t \omega^*(\mu_t)}{W_{t-1}}\right\}.$$

When $P_t < \varphi P_t^\star$, the DNWR constraint binds and raising P_t reduces real wages (encouraging hiring, reducing unemployment) without any effect on nominal wage inflation. On the other hand, when $P_t \geq \varphi P_t^\star$, the DNWR does not bind. In that case, an increase in P_t has no effect on real wages (and thus on unemployment) but translates one-for-one into higher nominal wage inflation. Thus, the price level $P_t = P_t^\star$ implements the lowest possible level of real wages (and unemployment) while keeping nominal wage inflation at zero. Our labelling of this policy as "conventional" reflects its similarity to interest rate rules studied in the NK literature, as well as its closeness to the US Federal Reserve's policy framework.[15]

The interest rate rule (15) indicates that monetary policy raises rates whenever prices exceed their target $P_t^\star$, and cuts rates when prices undershoot their target, subject to the ZLB. It is convenient to assume a very strong monetary response, i.e., $\phi_\omega \to \infty$ in equation (15), implying that in equilibrium:

$$P_t \leq \frac{W_{t-1}}{\omega^*(\mu_t)}, \qquad i_t \geq 0, \qquad \text{with at least one equality.} \tag{16}$$

In words, either the ZLB does not bind and prices are at their target level, or the ZLB binds and prices are below target. In what follows, we refer to the interest rate rule (15) with $\phi_\omega \to \infty$ as CMP. Importantly, as shown in Appendix F, CMP implements the same allocations as would be chosen by a monetary policymaker acting optimally under discretion.[16]

Given initial conditions μ_0 and W_{-1}, the equilibrium under CMP is a sequence $\{W_t, P_t, i_t, J_t, \omega_t, \theta_t, \mu_{t+1}\}_{t=0}^\infty$ satisfying (6), (7), (10), (12), (14), (16) and $\omega_t = W_t/P_t$ for all $t \geq 0$. To understand the economy's response to a demand shock, it is useful to note that given the wage setting rule (12) and the policy rule, nominal wages never increase in equilibrium even though they are fully flexible upwards. For ease of exposition, we focus below on the case with $\varphi = 1$ (nominal wages cannot fall).[17] In that case, equilibrium nominal wages are constant ($W_t = W_{-1}$ for all t), so the path of the real wage directly reflects that of the price level.

[15] Like the monetary authority in our model which aims to implement zero wage inflation and to keep unemployment at its "natural" flexible wage level, the Fed has had targets for both prices and real activity. In terms of prices, the Fed targets price inflation rather than nominal wage inflation; in our model, it makes more sense for the monetary authority to target nominal wages since these are the prices that are sticky (Aoki, 2001).

[16] Appendix F shows that this policy is optimal under discretion for a planner who minimises $(u_t - 0)^2 + \lambda \left(\frac{W_t}{W_{t-1}} - 1\right)^2$, where λ is the relative weight the planner puts on stabilising nominal wage inflation. While this objective function is not explicitly derived from household welfare, it is meant to reflect central banks' preference for stable inflation in addition to full employment, which many alternative models of nominal rigidities would feature (see, e.g., Woodford, 2003). The inflation stabilisation motive need not be large for the result to hold: CMP is the solution to the discretionary planner's problem for any $\lambda > 0$, however small.

[17] At the end of this section, we show that our characterisation of the equilibrium dynamics under CMP is unaffected by this restriction.

Effects of a transitory discount factor shock Assuming that the economy is initially at the full employment steady state ($\mu_0 = 0$), we now describe its response to a transitory demand shock ($\beta_0 > 1$, $\beta_t = \beta < 1$ for all $t > 0$) under CMP. This shock causes the ZLB to bind and causes the date 0 price level to fall. Since nominal wages cannot fall, real wages increase, discouraging firms from posting vacancies. For small shocks, while vacancy creation falls, the economy remains at full employment. Larger shocks cause unemployment to rise; since unemployment erodes human capital, this increases the fraction of unskilled job seekers μ_1, driving the economy into the convalescent region and resulting in a slow recovery. If the shocks are larger still, μ_1 may enter the stagnant region, resulting in permanent stagnation. These dynamics are summarised in Proposition 4.

Proposition 4 (Conventional monetary policy)**.** *Assuming that the economy is initially at the full-employment steady state* ($\mu_0 = 0$)*, there exists a* $\underline{\beta} = \frac{A-\kappa}{\omega^*_{\text{fe}}-(1-\delta)J_{min}} > 1$ *such that:*

1. *If the shock is not too large,* $\beta_0 \in (1, \underline{\beta}]$*, vacancy posting falls,* $\theta_0 \in (1, \theta^{\text{fe}})$*, but the economy remains at full employment,* $\mu_t = 0$ *for all* t*.*

2. *For a large enough shock,* $\beta_0 > \underline{\beta}$*, labour markets are slack,* $\theta_0 \in [0, 1)$*, and unemployment rises,* $\mu_1 \in (\underline{\mu}, \mu_R]$*, where* $\mu_R = (2-\delta)^{-1}$ *is the rate of skill depreciation after a one-period hiring freeze, starting from full employment.*

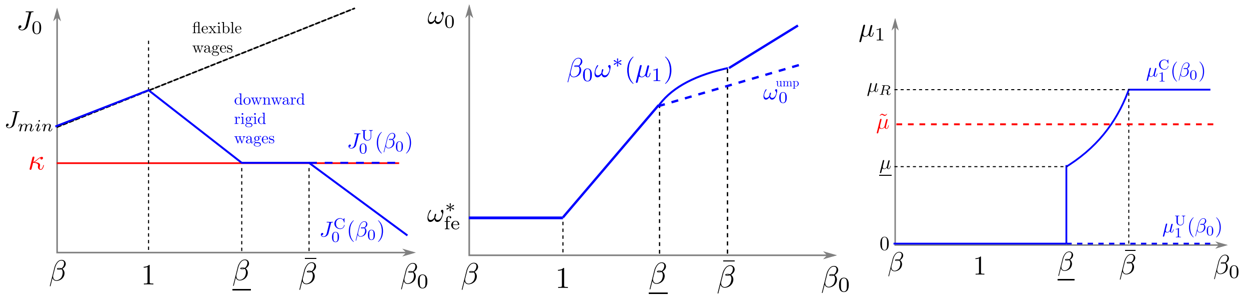
Furthermore, for $\beta_0 > \underline{\beta}$*, if* $\mu_1 < \widetilde{\mu}$*, then the economy eventually returns to full employment whereas if* $\mu_1 \geq \widetilde{\mu}$*, then the economy never returns to full employment.*

Proof. See Appendix H. □

In the flexible wage benchmark, a temporary increase in β_0 would not raise unemployment. In fact, since a filled vacancy is a long-lived asset yielding dividends in the future and the cost of posting a vacancy is paid today, a temporary increase in the discount factor increases the net present value of vacancy posting, *encouraging* vacancy creation (a *neoclassical effect*). However, with nominal rigidities, under CMP, outcomes differ from the flexible wage benchmark economy when the ZLB binds. To be clear, this does not mean that it is impossible for *any* monetary policy to replicate the flexible wage outcome when the ZLB binds - in the next section, we will study an unconventional monetary policy that does exactly that. It only means that *conventional* monetary policy fails to do this.

All else equal, a higher β_0 increases households' demand for bonds which lowers the demand for current consumption and puts downward pressure on its price P_0. CMP tries to prevent P_0 from falling by lowering the nominal rate, dissipating the excess demand for bonds. When the ZLB binds, the nominal return on bonds cannot be lowered any further and the Fisher equation (14) indicates that inflation between dates 0 and 1 must satisfy $P_1/P_0 = \beta_0 > 1$. Whether this requirement is met in equilibrium by a rise in P_1 or a fall in P_0 depends on the conduct of monetary policy.

Under CMP, P_1 never rises enough to prevent a fall in P_0 in equilibrium: P_0 must fall in response to a demand shock of magnitude $\beta_0 > 1$. Accordingly, real wages and the cost of hiring at date 0 rise with the decline in P_0. To see why P_0 must fall, suppose that it instead remains constant at P_{-1}. Since the ZLB binds at date 0, this implies that P_1 must rise to $P_1 = \beta_0 P_{-1}$. Accordingly, real wages



(a) Value of a vacancy **(b)** Date 0 real wages **(c)** Skill composition of job-seekers

Figure 3. Response to a demand shock: Panel (a) - the solid non-monotonic curve depicts the relationship between J_0 and β_0 under CMP, the flat dashed line represents the same relationship under UMP, the upward-sloping dashed line represents the same relationship absent nominal rigidities and the flat line at $J_0 = \kappa$ represents the cost of job-creation at date 0. Panel(b) - the solid curve depicts the relationship between ω_0 and β_0 under CMP while the dashed curve represents same relationship under UMP. Panel (c) - the solid curve represents the relationship between μ_1 and β_0 under CMP while the dashed line represents the same relationship under UMP.

remain at ω^*_{fe} at date 0 and 1, implying that nominal wages increase between date 0 and 1, $W_1/W_0 > 1$. But since the ZLB does not bind at date 1, CMP implements $P_1^\star = W_0/\omega^*_{\mathrm{fe}}$ (see equation (16)). This, however, implies that $W_1/W_0 = 1$, which is a contradiction. Thus, P_0 must fall. Intuitively, because CMP aims to implement a price level consistent with zero nominal wage inflation at date 1, P_1 cannot rise enough to prevent P_0 from falling in equilibrium.

Since nominal wages cannot fall, a lower P_0 raises real wages at date 0. Thus, when the ZLB binds, the neoclassical effect of a higher β_0 can be outweighed by a deflationary *Keynesian effect*, causing a fall in the value of vacancy creation. With perfectly flexible nominal wages, only the neoclassical effect would operate, generating a positive relationship between the value of a filled vacancy J_0 and size of the shock β_0, as shown by the dashed upward sloping line in Figure 3a. In contrast, with nominal rigidities, the Keynesian effect is also at work, and under Assumption 2, it always dominates the neoclassical effect, resulting in a negative relationship between J_0 and β_0 whenever $\beta_0 > 1$, as shown by the solid downward sloping line in Figure 3a.

As stated in Proposition 4, the ultimate effect of a higher β_0 on the labour market depends on the size of the shock. Figures 3a, 3b and 3c depict the date 0 value of a filled vacancy, the date 0 real wage, and the date 1 fraction of unskilled job seekers, respectively, as functions of β_0. We next turn to a detailed discussion of these relationships.

Moderate shocks When the increase in β_0 is moderate $\beta_0 \in (1, \underline{\beta}]$, while real wages increase and the value of a filled vacancy falls, this value does not fall enough to reduce vacancy posting substantially. Consequently, the economy remains at full employment ($\mu_1 = 0$). This means that under CMP, $P_1 = W_{-1}/\omega^*_{\mathrm{fe}}$. The binding ZLB at date 0 then implies that P_0 falls to $P_1/\beta_0 < P_{-1}$, and the real wage rises to $\omega_0 = \beta_0\omega^*_{\mathrm{fe}} > \omega^*_{\mathrm{fe}}$ (see Figure 3b). This reduces the value of a filled vacancy:

$$J_0 = \underbrace{A - \beta_0\omega^*_{\mathrm{fe}}}_{\text{current profit}} + \underbrace{\beta_0(1-\delta)J_{min}}_{\text{valuation of future profits}} . \tag{17}$$

The two opposite effects of a higher β_0 onto the value of a filled vacancy are visible in (17): a higher real wage lowers current profits (Keynesian effect) but a higher β_0 increases the valuation of future profits (neoclassical effect). Appendix G shows that under Assumption 2, the Keynesian effect dominates, i.e., $\partial J_0/\partial\beta_0 = -\omega^*_{\text{fe}} + (1-\delta)J_{min} < 0$. For $\beta \in (1, \underline{\beta}]$, the value of a filled vacancy J_0 falls with β_0, but remains above the vacancy posting cost κ (see Figure 3a), implying that firms are still willing to post enough vacancies to keep the economy at full employment ($\mu_1 = 0$), as shown in Figure 3c.

Large shocks When the shock is larger, $\beta_0 > \underline{\beta}$, the real wage at date 0 increases by so much that the value of a filled vacancy either falls to or below the vacancy posting cost κ. Firms are not willing to post vacancies if they expect full employment (and thus high wages) to prevail in the future. Unemployment must thus rise, along with a worsening in the skill composition, for future wages to be low enough to possibly convince firms to keep hiring at date 0. To see this, note that the date 0 value of a filled vacancy is now given by

$$J_0 = \underbrace{A - \beta_0\omega^*(\mu_1)}_{\text{current profit}} + \underbrace{\beta_0(1-\delta)(\kappa + \chi\mu_1)}_{\text{valuation of future profits}} \quad \text{where } \mu_1 \in [\underline{\mu}, \mu_R]. \tag{18}$$

For firms to post vacancies at date 0 despite a high β_0, future profits must rise just enough to make the value of a filled vacancy J_0 in (18) at least as large as the vacancy posting cost κ. For future profits to rise, the fraction of unskilled job seekers must be high enough to generate low real wages even after the shock has abated: the economy enters the convalescent or stagnant region, i.e., $\mu_1 > \underline{\mu}$ (see Figure 3c). This rise in μ_1 means that CMP targets a higher price level $P_1^\star = W_0/\omega^*(\mu_1)$, which implies that P_0 falls by less from the Fisher equation. This mitigates but does not wholly prevent the rise in date 0 real wages, as shown by the solid curve between $\underline{\beta}$ and $\overline{\beta}$ in Figure 3b.

For $\beta_0 \in (\underline{\beta}, \overline{\beta}]$, the rise in μ_1 increases future profits enough to offset the decline in current profits, leaving $J_0 = \kappa$ in this region (flat segment of the solid non-monotonic curve between $\underline{\beta}$ and $\overline{\beta}$ in Figure 3a). However, for extremely large demand shocks ($\beta_0 > \overline{\beta}$) the value of the firm J_0 falls below κ, and firms are no longer willing to post new vacancies - the hiring rate falls to zero and the date 1 fraction of unskilled job seekers increases to μ_R.[18] While this rise in μ_1 increases future profits, it is not enough to offset the fall in current profits and prevent J_0 from falling below κ. Note that while firms are unwilling to post new vacancies when $J_0 < \kappa$, they do not wish to destroy existing jobs, which also pay higher real wages, as long as $J_0 > 0$. This is true in all the scenarios we consider. That is, nominal rigidities are never so severe that they drive the real wage out of the bargaining set. Since existing matches still observe positive surplus, firms have no incentive to shut down and employed workers have no incentive to agree to wage cuts. Workers and firms do not forgo mutually beneficial wage cuts and the Barro (1977) critique does not apply.

In the event of a large increase in unemployment, the economy experiences either a slow recovery or a permanent stagnation. Which of these scenarios realises depends both on the size of the shock and

[18]See Appendix H for the definition of $\overline{\beta}$. In our economy with only one period shock, μ_R is the maximum damage that can be inflicted on the skill composition of the workforce during the ZLB episode, starting from full employment. Shocks which last longer could of course result in a higher μ.

on the forces generating multiple steady states. The higher the training cost χ, the lower the threshold $\widetilde{\mu}$, and the more likely it is that the economy enters the stagnant region following a sufficiently large shock.[19] Similarly, for a given χ, a larger β_0 is more likely to push the economy into the stagnant region. Figure 3c depicts the case in which $\mu_R > \widetilde{\mu}$, so a large enough shock that causes a hiring freeze at date 0 always drives the economy to the stagnant region.

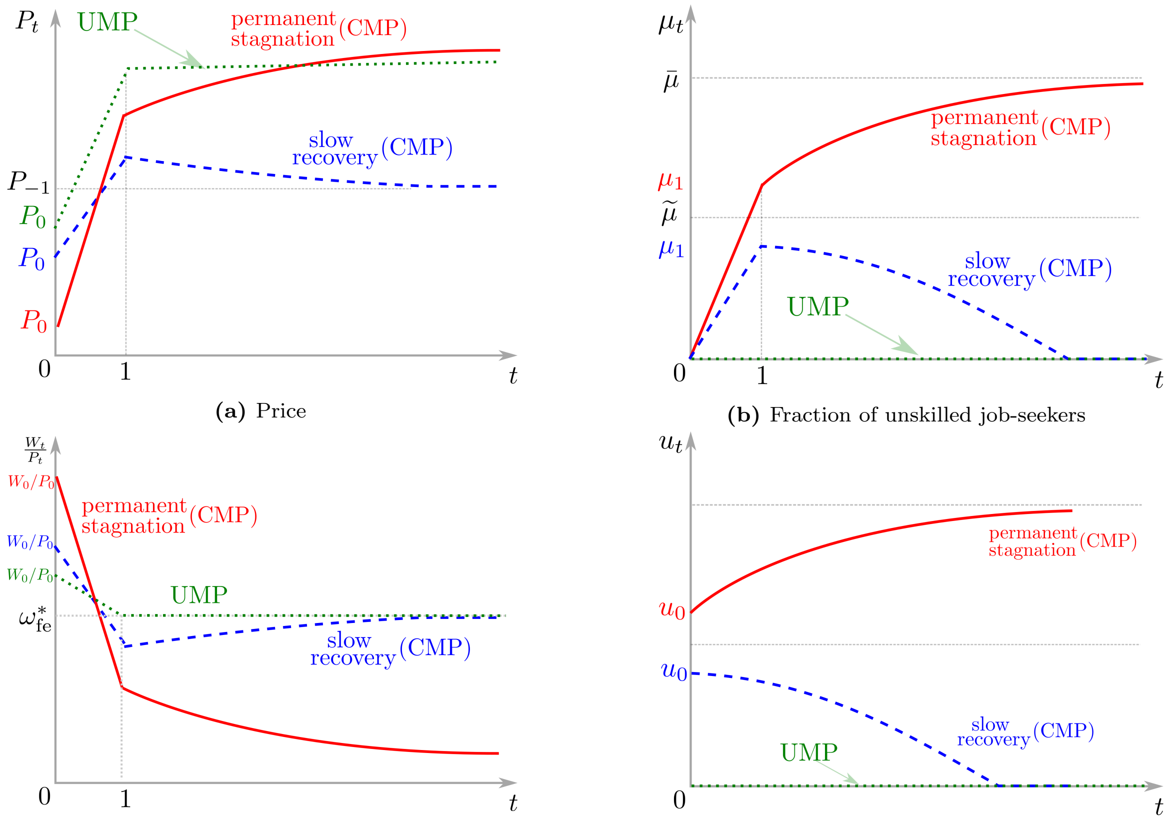
Slow recovery When the reduction in hiring drives the economy into the convalescent region, i.e., when $\mu_1 \in (\underline{\mu}, \widetilde{\mu})$, the economy ultimately returns to full employment, but the recovery takes time. The dashed line in Figure 4b depicts the dynamics of μ following a shock which drives the economy to the convalescent region. Following an initial deterioration in the skill composition of the workforce due to a hiring slump, the economic forces underlying this slow recovery are essentially the ones outlined in Section 3. Faced with a higher likelihood of meeting unskilled applicants and hence higher expected training costs, firms only post vacancies if they are compensated by lower real wages. In turn, the only way low wages can be an equilibrium outcome is if job-finding rates are depressed for some time, keeping the worker's outside option low. As a result, the unemployment rate and the fraction of unskilled job-seekers only decline gradually, but the economy ultimately returns to full employment.

Permanent stagnation When the date 0 hiring slump takes the economy into the stagnant region ($\mu_1 \geq \widetilde{\mu}$), the economy never returns to full employment. The solid line in Figure 4b shows the dynamics of μ in this case. Again, conditional on the initial deterioration of the skill composition, the forces behind the ensuing stagnation dynamics are not nominal but real – the DNWR constraint does not bind beyond date 0. Unemployment remains permanently high *not* because of nominal frictions but because of a deterioration of unemployed workers' human capital. The fraction of unskilled job-seekers is so high that real wages must be very low for firms to post *any* vacancies. Such low real wages can only be sustained if slack labour markets are expected to persist forever, implying that the economy converges to the high unemployment steady state. In this steady state, even though high unemployment depresses wages, firms are reluctant to post vacancies because the average job-seeker is likely to be unskilled and costly to retrain. These low vacancy posting rates support high unemployment. Thus, even a transitory demand shock can permanently depress employment.

Dynamics of prices, wages and unemployment After date 1, the evolution of the price level depends on whether the economy eventually returns to full employment, or converges to the high unemployment steady state. If the economy returns to full employment, the real wage eventually returns to ω^*_{fe} (as shown in Figure 4c), and the (targeted and realised) price level falls back to P_{-1} (shown by the dashed line in Figure 4a). If instead the economy converges to the high unemployment steady state, the real wage falls further to $\omega^*(\overline{\mu})$, and the price level rises further to $W_{-1}/\omega^*(\overline{\mu})$ (solid line in Figure 4a). Regardless of the scenario, the price level rises above its pre-shock value at date 1

[19] More generally, shocks lasting multiple periods would also be more likely to bring the economy to the stagnant region. In this section we focus on one period shocks to emphasise that even very transitory recessions can have permanent effects. However, in our quantitative analysis in Online Appendix L, we allow for persistent shocks.

when $\beta_0 > \underline{\beta}$, but does not rise enough to prevent unemployment at date 0 (as shown in Figure 4d). The unconventional monetary policy we propose in Section 4.2 does not share this shortcoming.



(a) Price

(b) Fraction of unskilled job-seekers

(c) Real Wages

(d) Unemployment

Figure 4. Dynamics under CMP and UMP: Panel (a) depicts the trajectory of the price level, panel (b) depicts the trajectory of μ, panel (c) depicts trajectory of real wages and panel (d) depicts the trajectory of the unemployment rate. In all panels, the solid curves depict the case where the economy features permanent stagnation under CMP, the dashed lines depict the case where the economy features a slow recovery under CMP and the green dotted lines depict the case under UMP.

4.1.1 Case with $\varphi < 1$

Before discussing how unconventional monetary policy can prevent hysteresis from taking root, it is useful to highlight that our results do not depend on nominal wages being fully rigid downward. The discussion above assumed $\varphi = 1$, i.e., nominal wages cannot fall however large the shock or level of unemployment. Nonetheless, Proposition 4 remains true even if we allow nominal wages to fall by some amount ($0 < \varphi < 1$) and even if this degree of flexibility is state dependent, as long as nominal wages are not fully flexible ($\varphi = 0$). In particular, the dynamics of all real variables remain the same as described above even if nominal wages are allowed to fall somewhat.

With $\varphi < 1$, following a shock $\beta_0 > 1$, nominal wages do fall by the maximum amount possible to $W_0 = \varphi W_{-1}$. While in partial equilibrium, such a fall in nominal wages would encourage firms to post vacancies, this increased incentive to post vacancies is negated by general equilibrium forces – prices fall by a larger amount, leading real wages to rise to the exact same level as when $\varphi = 1$. To see why, recall that since the ZLB does not bind at date 1, the CMP implements $P_1 = P_1^\star = W_0/\omega^*(\mu_1)$. However, the ZLB does bind at date 0, so the Euler equation implies

$$P_0 = \frac{1}{\beta_0} P_1 = \frac{1}{\beta_0} \frac{W_0}{\omega^*(\mu_1)}.$$

Thus, the real wage satisfies $\omega_0 = \beta_0 \omega^*(\mu_1)$ and the date 0 value of a filled vacancy is given by $J_0 = A - \beta_0 \omega^*(\mu_1) + \beta_0(1-\delta)J_1$, exactly as in the $\varphi = 1$ case. And since the ZLB and DNWR constraints do not bind after 0, all remaining equations characterising the dynamics of the real variables are identical to the ones obtained when $\varphi = 1$, and the proof of Proposition 4 equally applies (see Online Appendix J). Thus, allowing the date 0 nominal wage W_0 to adjust downwards by some degree triggers a proportional decline in the date 0 price level P_0, resulting in the real wage ω_0 remaining the same as when $\varphi = 1$. Therefore, greater nominal wage flexibility does not mitigate the rise in real wages.

Intuitively, CMP anchors all future nominal variables to the date 0 nominal wage. Therefore, any additional drop in the date 0 nominal wage is accompanied by a fall of the same magnitude in nominal variables at date 1 and beyond. But by targeting a lower price level P_1 at date 1, the monetary authority further exacerbates the decline in households' consumption demand at date 0. Accordingly, date 0 prices must fall even further so as to dissipate excess demand on the date 0 bond market, leaving real wages unchanged relative to the case where $\varphi = 1$.

Since this argument was made for any $\varphi \in (0,1)$, it also explains why our results remain unchanged if the degree of nominal wage flexibility φ is state-dependent, as Online Appendix J shows.

4.2 Response under Unconventional Monetary Policy (UMP)

CMP arguably provides a good description of the conduct of monetary policy in many advanced economies (at least prior to the Great Recession), and – as already mentioned – it is optimal under discretion (see Appendix F). However, as we have shown, this common approach to policy can lead to undesirable outcomes in the presence of hysteresis.

The problem with CMP is that when the ZLB binds in the present, policy fails to commit to more expansionary policy in the future to forestall deflation today. As we now show, an *unconventional monetary policy* (UMP) which makes such commitments mitigates deflationary forces at date 0 and prevents the initial rise in unemployment and the subsequent damage, even for large shocks. Similar to CMP, we specify UMP as attempting to implement a time-varying price level target $\{\widetilde{P}_t^\star\}_{t=0}^\infty$. In particular, when the initial shock β_0 is small ($\beta_0 \leq \underline{\beta}$), then the price level target $\widetilde{P}_t^\star$ under UMP is identical to the target $P_t^\star$ under CMP. However, when the shock is larger than $\underline{\beta}$, UMP is more accommodative and commits to a higher price level target at date 1 ($\widetilde{P}_1^\star > P_1^\star$) and beyond. Specifically, for $\beta_0 > \underline{\beta}$, $\{\widetilde{P}_t^\star\}_{t=0}^\infty$ is given by:

$$\widetilde{P}_0^\star = \frac{W_{-1}}{\omega_0^{\text{ump}}(\beta_0)} \qquad \text{and} \qquad \widetilde{P}_t^\star = \beta_0 \widetilde{P}_0^\star \qquad \text{for all } t \geq 1 \tag{19}$$

where $\omega_0^{\text{ump}}(\beta_0) = A - \kappa + \beta_0 (1-\delta) J_{min}$ is the real wage which sets the value of a filled vacancy at date 0 to κ ($J_0 = \kappa$). Note that since $\omega_0^{\text{ump}}(\beta_0) > \omega_{\text{fe}}^*$, we have $\widetilde{P}_0^\star < P_{-1} = W_{-1}/\omega_{\text{fe}}^*$, so this policy permits some deflation at date 0 – but crucially, *less* deflation than observed in equilibrium under CMP. Proposition 5 describes how outcomes differ under UMP when the economy experiences a shock β_0 large enough that it would move away from full employment under CMP.

Proposition 5 (Unconventional monetary policy)**.** *Suppose that $\beta_0 > \underline{\beta}$ and the economy starts at full employment, i.e., $\mu_0 = 0$ and $W_{-1} = \omega_{\text{fe}}^* P_{-1}$. Under UMP, the economy remains at full employment, $\mu_t = 0$ for all $t \geq 0$. Prices are equal to target at all dates $t \geq 0$, $P_t = \widetilde{P}_t^\star$ for $\widetilde{P}_t^\star$ described in (19). Nominal wages are given by $W_0 = W_{-1}$ and $W_t = \beta_0 \frac{\omega_{\text{fe}}^*}{\omega_0^{ump}(\beta_0)} W_{-1} > W_0$ for all $t \geq 1$. Real wages are given by $\omega_0 = \omega_0^{ump}(\beta_0)$ and $\omega_t = \omega_{\text{fe}}^*$ for all $t \geq 1$.*

Proof. See Appendix I ☐

Recall that under CMP, the ZLB binds at date 0 and the price level P_0 falls to ensure that $P_1/P_0 = \beta_0 > 1$. While the future price level P_1 does rise under CMP, it does not rise enough to prevent a large fall in current prices P_0, which raises real wages and reduces hiring. In contrast, UMP commits to implement a date 1 price level high enough to prevent date 0 prices from falling to a level which would reduce hiring. While a large β_0 shock increases the demand for savings, a higher future price level P_1 attenuates this increase by making future consumption more expensive. The resulting higher demand for date 0 consumption hence mitigates the fall in P_0. Thus, while the date 0 real wage does rise under UMP, it only rises to $\omega_0^{\text{ump}}(\beta_0)$ (shown by the dashed line in Figure 3b). As a result, the value of a filled vacancy J_0 never falls below the cost of posting a vacancy κ, however large β_0 is (dashed line in Figure 3a). Firms are willing to post enough vacancies to keep the economy at full employment despite the fall in prices. And since μ_1 does not rises above 0 (dashed in Figure 3c), the economy never enters the convalescent or stagnant regions and hysteresis is averted. However, this policy comes at the cost of positive nominal wage inflation between dates 0 and 1. The time paths of the price level, the fraction of unskilled job-seekers, the real wage and rate of unemployment are represented by the dotted green lines in Figures 4a, 4b, 4c and 4d, respectively.

UMP is similar to *forward guidance* policies (Eggertsson and Woodford, 2003; Werning, 2011) in that when the ZLB binds, it commits to more accommodative policy in the future. However, while forward guidance is usually perceived as a commitment to lower nominal rates in the future, UMP involves a commitment to target higher prices at date 1, but does not necessarily involve a lower path of nominal rates compared to CMP. Section 4.1 described how CMP can be thought of as a limiting case of a Taylor type rule (15) which raises rates aggressively when prices exceed their target, and cuts rates aggressively when prices undershoot their target, until nominal rates hit the ZLB. UMP can be thought of in the same way, but with a different price level target. Following a large shock at date 0, UMP targets a higher date 1 price level than CMP ($\widetilde{P}_1^\star > P_1^\star$). Off-equilibrium, UMP would cut interest rates more aggressively than CMP at date 1, if faced with the same observed price level P_1. On equilibrium, however, prices do not actually deviate from target at date 1 (since the ZLB does not bind at date 1) under either CMP or UMP. Thus, it is not necessarily the case that the unconventional policymaker implements lower nominal rates at date 1 in equilibrium.[20]

[20]In our economy with linear utility – and hence exogenous real interest rates, the nominal interest rate behaves

5 Discussion

5.1 Commitment vs discretion and the importance of timely accommodation

The difference between CMP and UMP is not that the ZLB binds under one policy and not the other: the ZLB also binds under UMP at date 0. Instead, the key difference is that UMP makes commitment regarding future policy. Indeed, just as CMP is optimal under discretion for a planner who minimises the loss function $u_t^2 + \lambda \left(W_t/W_{t-1} - 1\right)^2$, UMP is optimal under commitment for a planner with the same preferences, provided that λ is sufficiently small (see Appendix F).[21] Thus, one interpretation of our results is that a discretionary policymaker cannot prevent adverse shocks from causing hysteresis. This is because the policy required to avoid hysteresis is a timely commitment to more accommodative policy in the future (date 0 commitment to a high enough P_1).

However, the ability to commit is by itself not enough to overcome hysteresis. It is essential that commitments be made in a *timely* fashion, before unemployment has increased and the skill composition of job-seekers has deteriorated. Suppose that monetary policy fails to make such commitments at date 0 and is expected to follow CMP at all future dates, allowing unemployment to rise and pushing μ_1 into the stagnant region. Can policy then reverse course and return the economy to full employment - e.g. by committing at date 1 to implement the price sequence $\{\widetilde{P}_t^\star\}_{t=1}^\infty$ from then onward? The answer is no: monetary policy cannot engineer an escape from an unemployment trap. In the stagnant region, employers are unwilling to create more vacancies despite prevailing low real wages since the fraction of unskilled job seekers is large. If monetary policy could temporarily drive down real wages further, this would encourage hiring and bring down the unemployment rate. Recall, however, that our model features asymmetric nominal wage rigidities, in line with the empirical evidence. Nominal wages are rigid downwards, but flexible upwards: implementing a higher price level would only cause nominal wages to rise one-for-one, leaving real wages unchanged at their natural level. Similarly, in the event of a slow recovery rather than a permanent stagnation, monetary policy cannot speed up the recovery after date 0 if it was insufficiently accommodative to begin with: lower real wages would in principle stimulate hiring, but monetary policy cannot reduce real wages. This highlights that slow recoveries and permanent stagnation in our economy are *not* driven by nominal rigidities and a binding ZLB after date 0. They are driven by the elevated fraction of unskilled job seekers, a purely real factor. Only at date 0, when the DNWR binds and deflation drives real wages above their natural level, can monetary policy improve outcomes by mitigating the deflationary pressures. *Timely commitment*, rather than commitment per se, is needed to prevent hysteresis.

The point that commitment delivers better outcomes than discretion, particularly when the ZLB binds, is well known in the NK literature (see, e.g., Werning 2011). However, our emphasis on *timely* commitments is novel, relative to this literature. While commitment to expansionary future policy is

analogously to expected inflation. In the slow recovery scenario, prices decline after date 1 under CMP (Figure 4a), so expected inflation and nominal rates are lower than under UMP, which features constant prices after date 1. In contrast, in the permanent stagnation scenario, prices rise after date 1 under CMP, so nominal rates are higher under CMP than under UMP.

[21] While UMP involves some wage inflation at date 1, it avoids persistently or permanently higher unemployment, which outweighs the cost of inflation provided that the planner does not put too high a weight on stabilising inflation.

effective in standard NK models, it is equally effective at any point in the recession. Further, delaying monetary accommodation in such models is costly, but the costs are only temporary. In contrast, delayed commitments are ineffective in our model, and a failure to make a timely commitment can have permanent costs if shocks are large.

5.2 Is timeliness generally important?

A timely response of monetary policy is of paramount importance in our baseline model, since if monetary policy fails to act at date 0, it is powerless to speed up a recovery or escape an unemployment trap at a later point. This result depends crucially on two special assumptions in our baseline model. First, nominal wages are flexible upwards, implying that monetary policy cannot push real wages below their natural level by reducing prices to stimulate hiring after hysteresis has taken root. Second, we have abstracted from other policies, such as hiring and training subsidies, which could help bring the economy back to full employment. Thus, one might wonder whether timeliness would be as important if nominal wage rigidities were symmetric or if other fiscal policy instruments were available. As we show next, either symmetric wage rigidities or fiscal policy make it possible for policy to accelerate recoveries or escape traps ex post. However, doing so remains unattractive, as it involves higher costs than what would have been incurred under a timely monetary policy response preventing any increase in unemployment in the first place.

5.2.1 Symmetric nominal rigidities

Suppose that instead of being downwardly rigid (as suggested by the empirical evidence reviewed in our introduction), nominal wages were fixed at $\overline{W}$, arguably the simplest form of a symmetric rigidity. UMP at date 0 would again prevent any increase in unemployment. However, if an alternative policy allowed a large shock to push the economy into the convalescent or stagnant region, monetary policy would be able to speed up a recovery or escape from an unemployment trap at a later date, unlike under DNWR. With a fixed nominal wage, implementing persistently higher prices from date 1 onwards would lower real wages and encourage hiring even when the skill composition of job-seekers make expected training costs high. Such a strategy, however, entails larger losses than enacting UMP at date 0.

Specifically, since nominal wages are fixed in this case, we specify the planner's loss function in terms of unemployment and price (rather than wage) inflation: $u_t^2 + \lambda_p \left(P_t/P_{t-1} - 1\right)^2$.[22] Allowing unemployment to rise at date 0 and acting to restore full employment at date 1 entails a higher loss than following UMP from date 0 onwards. Under UMP, there is no unemployment, a modest deflation at date 0 and a modest inflation at date 1, yielding a loss of

$$\mathcal{L}_0^{\text{ump}} = \lambda_p \left(\frac{\widetilde{P}_0^{\star}}{P_{-1}} - 1 \right)^2 + \beta_0 \lambda_p \left(\frac{\widetilde{P}_1^{\star}}{\widetilde{P}_0^{\star}} - 1 \right)^2 = \lambda_p \left(\frac{\omega_{\text{fe}}^{*}}{\omega_0^{\text{ump}}} - 1 \right)^2 + \beta_0 \lambda_p \left(\beta_0 - 1\right)^2 .$$

[22] Again, while this objective function is not explicitly derived from household welfare, it is meant to reflect central banks' preferences for low unemployment and stable inflation.

For concreteness, consider the following alternative policy: CMP is initially expected to apply from date 0 onwards, but the policymaker unexpectedly deviates from CMP at date 1 and instead implements a higher than expected price level so as to lower real wages and bring the economy back to full employment. At date 0, this policy acts exactly like CMP, thus clearly entailing higher deflation and unemployment than UMP (see Figure 4).[23] At date 1, while inflation equals $\beta_0 > 1$ under UMP (and CMP), it is higher than β_0 under the alternative policy, which implements a higher price than CMP from date 1 onwards to reduce real wages below $\omega^*(\mu)$ and bring the economy back to full employment. Thus, the alternative policy involves strictly higher losses than UMP, both at date 0 and at date 1.

In our baseline model with DNWR, a failure to act at date 0 forces the monetary authority to accept higher unemployment at date 1 — once μ is higher, no amount of inflation can bring unemployment back immediately (or perhaps ever). With symmetric rigidities, it remains true that a failure to act at date 0 leaves monetary policy facing a less favourable inflation-unemployment trade-off than would have obtained under UMP. With inflation at $P_1/P_0 = \beta_0 > 1$, unemployment is positive; reducing unemployment to zero is not impossible but requires inflation above β_0. Thus, even with symmetric rigidities, timeliness is important, not because it is impossible to reverse the shock's effects, but because doing so is more costly than committing early to higher future prices. While the empirical evidence arguably suggests that nominal wage rigidities are better modelled as asymmetric rather than symmetric, the logic of our timeliness argument holds more generally.

5.2.2 Fiscal policy

Our baseline model focuses on monetary rather than fiscal policy, reflecting the reality that fiscal policy is often imperfect and slow to respond to a downturn, leaving monetary policy to be a first responder when it comes to countercyclical stabilisation. Enriching our baseline model with fiscal policy tools, such as hiring or training subsidies, could make the effects of hysteresis less severe. Indeed, compensating firms for each worker they train would be equivalent to lowering the private training cost χ, potentially speeding up a recovery or even lifting the economy out of the stagnant region. However, the fact that fiscal policy can make up for a failure of monetary policy to act early does not imply that fiscal policy is necessarily more appropriate to address hysteresis, nor does it make timeliness less of a relevant issue. Even in the presence of fiscal policy, an appropriately designed monetary policy such as UMP is capable of keeping the economy at full employment and preventing any skill depreciation. While hiring and training subsidies might be effective at mitigating or reversing an increase in unemployment, they can hardly do better in terms of employment outcomes than a monetary policy which keeps the economy at full employment throughout.

In addition, fiscal policies are generally not costless. If the fiscal authority were to introduce hiring or training subsidies to speed up the recovery following an initial increase in unemployment, this would entail lower aggregate output (due to unemployment), lower consumption due to the real resources which must be spent on training, and distortionary costs of taxation to finance the subsidy. These costs may well be higher than the costs of temporarily higher inflation associated with UMP. For fiscal policy to avoid these costs and deliver the same unemployment outcomes as UMP, it would be necessary

[23] Figure 4 shows $\mu_1 > 0$, which, given the one-to-one mapping between μ_t and u_{t-1}, implies $u_0 > 0$.

to subsidise job creation at date 0 before any rise in unemployment has occurred. Implementing such time-varying state-dependent subsidies could be quite demanding, both administratively and politically. Similar issues would arise if one attempted to prevent hysteresis using automatic stabilisers. Any policy which automatically increases hiring or training subsidies after a downturn would incur the same resource costs mentioned above. For automatic stabilisers to avoid such costs and be as attractive as UMP, they would need to subsidise hiring or training before any increase in unemployment has occurred. One extreme way to do this would be for the government to finance all training costs and charge firms a constant tax. Again, such a policy could involve substantial administrative costs (which are not captured by our model). Thus, timely monetary policy action does not immediately lose its appeal when fiscal policy is considered.

To reiterate, it is true in our model that once unemployment has reached a high level, it causes skill depreciation which monetary policy is ill-equipped to reverse. Such skill depreciation would be better addressed by labour-market specific policies such as hiring and training subsidies. But what monetary policy *can* do at least as well as fiscal policy in this environment is prevent unemployment from rising in the first place, which avoids any skill depreciation, obviating the need for ex-post labour-market policies. In more complicated models (i.e., models featuring more shocks and/or distortions), it may well be that neither monetary nor fiscal policy can address every distortion on its own. Analysing the optimal policy mix in such cases is a interesting avenue for future research.

5.3 Comparison with recent papers studying permanent stagnation

In our baseline model, we highlighted how even a shock that lasts for one period can permanently move the economy away from full employment. As such, our analysis shares some similarities with a number of recent studies, such as Benigno and Fornaro (2017) and Schmitt-Grohe and Uribe (2017) which explore the possibility of permanent stagnation. However, there are two key differences that distinguish our analysis from theirs.

First, in our economy, a binding DNWR constraint at date 0 is necessary for demand shocks to move the economy away from full employment, but once unemployment has increased, nominal rigidities no longer bind and skill depreciation during unemployment – a purely real factor – causes slow recoveries or permanent stagnation. In fact, given the one period shock we consider, our results would remain the same even if we assumed that nominal wages were fully flexible after the first period. In contrast, in Benigno and Fornaro (2017) and Schmitt-Grohe and Uribe (2017) a shock which drives the economy to the ZLB can cause permanently higher unemployment, but only if the ZLB and DNWR also bind forever. While monetary policy and demand shocks can have permanent effects on unemployment in our economy, this is not because of a long run Phillips curve through which permanent deflation causes permanently higher unemployment (Benigno and Ricci, 2011) – our DNWR does not bind in steady state. Instead, temporary deflation can generate permanently higher unemployment.

Second, our paper brings the idea of path dependence into the literature on liquidity traps and secular stagnation. Again, this yields starkly different predictions from Benigno and Fornaro (2017) and Schmitt-Grohe and Uribe (2017). These papers feature multiple equilibria, one of which features deflation and high unemployment.[24] In these models, persistent unemployment can be an equilibrium outcome because agents' pessimistic beliefs are self-reinforcing. If agents, however, awoke one morning and expected the economy to return to full employment, the economy would indeed return to full employment. In contrast, our economy is not trapped in the high unemployment steady state because of self-fulfilling beliefs. In fact, starting from this steady state, if (off equilibrium) firms anticipated a return to full employment, they would be less willing to hire workers today, since they would anticipate a more skilled workforce and lower costs of job creation tomorrow. Lack of hiring today would further cement the skill deterioration in the workforce and reinforce high unemployment rates. Thus, persistently high unemployment arises in our model not because of self-fulfilling beliefs, but because our economy features an endogenous, slow-moving state variable – the skill composition of job-seekers.

6 Conclusion

We presented a model designed to study the positive and normative implications of hysteresis. Skill depreciation, nominal rigidities and constraints on monetary policy together allow temporary shocks to generate slow recoveries or even permanent stagnation. Aggressive countercyclical policy may be able to avoid these outcomes, but only if enacted in a timely manner. While we have focused on skill depreciation, more generally recessions may damage productive capacity through multiple channels - reducing capital accumulation, reducing labour force participation, slowing productivity growth, and so on. Many of these effects may also be hard or even impossible to reverse. For example, Wee (2016) shows that recessions can permanently change young workers' search behaviour, causing them to stay in careers in which they have a comparative disadvantage but have accumulated sufficient specific human capital, causing permanent misallocation. Whenever such mechanisms are operative, it is all the more important for countercyclical policy to nip recessions in the bud; the damage from failing to do so may be irreversible. Several open questions remain. Which policies are the most effective at preventing hysteresis? Are some policies more robust than others when the precise source of hysteresis effects are unknown? We hope that future research will shed light on these pressing issues.

[24] Of course, these models also feature multiple steady states in the sense that the economy can stay in the bad or good equilibrium forever.

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Appendix

A Wages and Nash bargaining

The value of an employed worker and of an unemployed worker are defined by the recursions (2) and (3). Also, the value of a filled vacancy to a firm is given by equation (6). We can then define the surplus of a match between a worker and a firm as:

$$\mathcal{S}_t = J_t + \mathbb{W}_t - \mathbb{U}_t.$$

Wages are determined by Nash bargaining. Denoting workers' bargaining power by η, wages solve

$$\max_{w_t} J_t^{1-\eta}(\mathbb{W}_t - \mathbb{U}_t)^{\eta},$$

implying

$$\eta J_t = (1-\eta)(\mathbb{W}_t - \mathbb{U}_t).$$

Notice that the match surplus can be rewritten as:

$$\begin{aligned} \mathcal{S}_t &= J_t + \mathbb{W}_t - \mathbb{U}_t, \\ &= A - b + \beta(1-\delta)J_{t+1} + \beta(1-\delta)(1-q_{t+1})(\mathbb{W}_{t+1} - \mathbb{U}_{t+1}), \\ &= A - b + \beta(1-\delta)(1-q_{t+1})S_{t+1} + \beta(1-\delta)q_{t+1}J_{t+1}. \end{aligned}$$

Using the fact that $\mathbb{W}_t - \mathbb{U}_t = \eta S_t$ in the equation above, we have:

$$\omega_t = \eta A + (1-\eta)b + \beta(1-\delta)\eta q_{t+1}J_{t+1}.$$

B Existence of multiple steady states

Define:

$$\underline{\eta} = \max\left\{\frac{1-\beta(1-\delta)}{\beta(1-\delta)}, \frac{1-\delta}{2-\delta}, \frac{(1-\delta)(\kappa+\chi)-b}{(1-\delta)(\kappa+\chi)+a-b}\right\}. \tag{B.1}$$

Steady states μ satisfies $a/\{1-\beta(1-\delta)[1-\eta(1-\mu)]\} = \kappa + \chi\mu$. Dividing through by J_{min}, this becomes

$$(1-e\mu)^{-1} = k + x\mu, \tag{B.2}$$

where $k = \kappa/J_{min}$, $x = \chi/J_{min}$ and $e = \beta\eta(1-\delta)/[1-\beta(1-\delta)(1-\eta)]$. For future reference, we define $\underline{\chi} = e\left[2-k+2\sqrt{1-k}\right]$. Assumptions 1 and 2 impose that $k < 1$ and $(1-e)^{-1} > k + x$. Since $e \in (0,1)$, $(1-e\mu)^{-1}$ is an increasing, strictly convex function. Starting from $x = 0$, as we increase x, either the intersection of these two functions first occurs at $\mu \in (0,1)$, in which case a slightly higher x

would give us multiplicity, or the first intersection has $\mu \geq 1$. Consider the knife edge case in which the first intersection of these two curves is at $\mu = 1$. Then the curves must be tangent and equal to each other at $\mu = 1$, i.e.

$$\frac{e}{(1-e)^2} = k \qquad \text{and} \qquad \frac{1}{1-e} = k + x,$$

which implies $k = (1-2e)(1-e)^{-2}$.

In order to have multiple intersections in $(0,1)$, there must exist some $\mu \in (0,1)$ such that $(1-e\mu)^{-1} = k + x\mu$ and $e(1-e\mu)^{-2} > x$ (at the larger of the two intersections, this convex function must intersect the linear function from below). If $k < (1-2e)(1-e)^{-2}$, then this cannot be the case. A smaller k implies a larger x, increasing the slope of the linear function; $\mu < 1$ decreases the slope of the convex function. Thus, we must have $k > (1-2e)(1-e)^{-2}$.The assumption that $\eta \geq \underline{\eta}$ is sufficient (but not necessary) to ensure this, since it implies that $e > 0.5$. If this is true, and if x is just large enough that there is a single slack steady state, then (B.2), which is quadratic in μ, has a unique solution, i.e. its discriminant equals zero: $x^2 - 2e(2-k)x + e^2k^2 = 0$.

Considered as a function of x, this equation has two real solutions since its discriminant is positive: $4(e^2(2-k)^2 - e^2k^2) = 16e^2(1-k) > 0$. This will have two solutions x^*, the larger of which corresponds to $\mu \in (0,1)$. To see this, consider the following graphical argument. Fix e and $k < 1$ and start with $x = \infty$, so that the $k + x\mu$ line is vertical at $\mu = 0$. Then the two curves intersect at exactly one point, $\mu = 0$. Decreasing x rotates the straight line clockwise, increasing the smallest value of μ at which the two curves intersect from 0 to some positive number. Eventually, for low enough x, the straight line is tangent to the convex curve at some $\mu > 0$. Next, start with $x = 0$, so the straight line $k + x\mu$ is horizontal at k and intersects the convex curve at some $\mu = e^{-1}(1-k^{-1}) < 0$. Gradually increasing x rotates the straight line counter-clockwise, lowering the first value at which the curves intersect. For x large enough, the two curves are tangent at some $\mu < 0$. Clearly, the second case corresponds to a lower value of x. Thus, the larger value of x corresponds to the economically sensible case where $\mu \in (0,1)$. Choosing this value, we have

$$x^* = e(2-k) + \sqrt{e^2(2-k)^2 - e^2k^2} = e[2 - k + 2\sqrt{1-k}].$$

Thus there will be multiple steady states if $x > x^*$.

C Proof of Proposition 1

Suppose $\mu_0 = 0$. Then, note that $\mu_t = 0$ (which implies $n_t = 1$) is consistent with (10), since in the tight labour market regime $q_t = 1$, and $n_t = 1, \mu_{t+1} = 0$. Next we show that we cannot have $\theta_0 < \theta^{\text{fe}}$ given $\mu_0 = 0$. (Since $\theta^{\text{fe}} \geq 1$ by Assumption 1, this implies in particular that we cannot have $\theta_0 < 1$.) In any equilibrium, (7) must be satisfied:

$$J_{min} \leq a \sum_{t=0}^{\infty} \prod_{\tau=0}^{t} \beta(1-\delta)(1-\eta\min\{\theta_\tau, 1\}) \leq \kappa \max\{\theta_t, 1\},$$

where the first inequality holds because the LHS is decreasing in θ_τ. Since we know that $J_{min} > \kappa$ from Assumption 1, it is immediate that this inequality can only be satisfied if $\theta_t \geq \theta^{\text{fe}} \geq 1$. Finally, we show that we cannot have $\theta_0 > \theta^{\text{fe}}$. We have shown that

$$a \sum_{t=0}^{\infty} \prod_{\tau=0}^{t} \beta(1-\delta)(1-\eta \min\{\theta_\tau, 1\}) = \kappa\theta_0$$

in any equilibrium, and that this expression is satisfied by $\theta_t = \theta^{\text{fe}}$, $\forall t \geq 0$. If $\theta_0 > \theta^{\text{fe}}$, it follows that $\theta_t < \theta^{\text{fe}}$ for some $t > 0$. Let T be the first date at which this is true. Then up to that date, since the labour market has been tight, $\mu_T = 0$. This is a contradiction, since we have already shown that if $\mu_T = 0$, $\theta_T \geq \theta^{\text{fe}}$. It follows that the unique equilibrium has $\theta_t = \theta^{\text{fe}}$ for all $t \geq 0$. The proof for any $\mu_0 \in (\mu_0, \underline{\mu})$ is similar and follows from the fact that $q_0 = 1$ which implies that all workers are employed in period 0. Before characterising the case when $\mu = \underline{\mu}$, the following result is useful:

Lemma 1. *If $J_t = J_{min}$, then $q_{t+1} = 1$, i.e. $\theta_{t+1} \geq 1$ and $J_{t+1} = J_{min}$.*

Proof. We have $J_t = a + \beta(1-\delta)(1-\eta q_{t+1})J_{t+1}$. The only way to attain $J_t = J_{min}$ is $q_{t+1} = 1$ and $J_{t+1} = J_{min}$, since $q_{t+1} \leq 1$, $J_{t+1} \geq J_{min}$, and the expression is decreasing in q_{t+1} and increasing in J_{t+1}. □

For $\mu_0 = \underline{\mu}$, there exist a continuum of equilibria indexed by $\theta_0 \in [1-\underline{\mu}, 1]$. In all these equilibria, the value of an employed worker for a firm is given by J_{min}. To see this, notice that $J_0 \leq \kappa + \chi\underline{\mu}$ as long as labour markets are slack, $\theta_0 \leq 1$. In this case, by definition, $J_0 \leq J_{min}$ and by definition this relationship has to hold with equality. If labour markets are tight, $\theta_0 > 1$, then $J_0 = \kappa\theta_0 + \chi\underline{\mu} > J_{\min}$ since $\theta_0 > 1$. This is a contradiction since if $\theta_0 > 1$, $\mu_1 = 0$ from Lemma 1 and $J_0 = J_{min}$ from Lemma 1. Furthermore, from Lemma 1, it follows that $J_1 = J_{min}$ and $\theta_1 \geq 1$.

The contradiction above shows that $\theta_0 \leq 1$. We now need to show that $\theta_0 > 1 - \underline{\mu}$. Suppose that $\theta_0 < 1 - \underline{\mu}$. Then μ_1 is given by:

$$\mu_1 = \frac{1-\theta_0}{1+(1-\delta)[1-\theta_0-\underline{\mu}]} > \underline{\mu}.$$

This is a contradiction since

$$J_1 = J_{min} = \kappa + \chi\underline{\mu} < \kappa + \chi\mu_1,$$

which requires that $\theta_1 = 0$. Thus, we have shown that $\theta_0 \in [1-\underline{\mu}, 1]$. From (4) and the earlier part of this proof, it follows that $\mu_1 = (1-\theta_0)/\{1+(1-\delta)[1-\theta_0-\mu_0]\} \leq \mu_0$ and $\theta_1 = (J_{min} - \chi\mu_1)/\kappa \geq 1$. As mentioned in footnote 8, we select the equilibrium in which $\theta_0 = 1$ implying that $\mu_1 = 0$, $\theta_1 = \theta^{\text{fe}}$.

D Proof of Proposition 2

Definition 1. *Define the functions* $\Theta^1 : I^1 \to [0,1]$, $F^1 : I^1 \to \mathbb{R}_+$, $M^1 : I^1 \to \{\underline{\mu}\}$ *as:*

$$\begin{aligned}
\Theta^1(\mu_{T-1}) &:= 1 - \frac{\underline{\mu}}{1-(1-\delta)\underline{\mu}}(1-(1-\delta)\mu_{T-1}), \\
F^1(\mu_{T-1}) &:= \frac{1}{\chi}\left[a - \kappa + \beta(1-\delta)(1-\eta\Theta^1(\mu_{T-1}))(\kappa + \chi\mu_{T-1})\right], \\
M^1(\mu_{T-1}) &:= \underline{\mu},
\end{aligned}$$

where $I^1 = [\underline{\mu}, \mu^1]$ *and* $\mu^1 := F^1(\underline{\mu})$.

Intuitively, at any date t, for any $\mu_t \in I^1$, $\Theta^1(\mu_t)$ describes the job-finding rate that ensures that the economy reaches $\underline{\mu}$ at date $t+1$. $F^1(\mu_t)$ describes the unique value that μ_{t-1} can have in period $t-1$ such that $\mu_t \in I^1$ and also $\mu_{t+1} = \underline{\mu}$. In other words, given market tightness at date t, $\Theta^1(\mu_t)$, one can compute the value of a filled vacancy at date $t-1$ and zero and by no-arbitrage, this pins down the value of μ_{t-1} for which firms would have been willing to post the requisite number of vacancies. $M^1(\mu)$ is just a constant function which by definition describes where any $\mu \in I^1$ ends up.

Corollary 1. *It must be true that* $\mu^1 < \widetilde{\mu}$.

By the definition of μ^1, it must be true that

$$\begin{aligned}
\mu^1 &= \frac{1}{\chi}\left[a - \kappa + \beta(1-\delta)\left[1-\eta(1-\underline{\mu})\right](\kappa+\chi\underline{\mu})\right] \\
&< \frac{1}{\chi}\left[a - \kappa + \beta(1-\delta)\left[1-\eta(1-\widetilde{\mu})\right](\kappa+\chi\widetilde{\mu})\right]. \\
&= \widetilde{\mu}
\end{aligned}$$

Lemma 2. *For* β *sufficiently close to 1,* F^1 *is increasing in* μ *for* $\mu \in [\underline{\mu}, \widetilde{\mu})$

Proof. Since $F^1(\mu)$ is composed of constants and a concave part, it suffices to consider the concave polynomial $\xi(\mu) = \left[1 - \eta\Theta^1(\mu)\right](\kappa + \chi\mu)$. This function is increasing in μ for

$$\mu < \frac{1}{2}\left[\frac{\left(1-(1-\delta)\underline{\mu}\right)(1-\eta)}{\eta(1-\delta)\underline{\mu}} + \frac{1}{(1-\delta)} - \frac{\kappa}{\chi}\right]. \tag{D.1}$$

It is thus sufficient to show that $\widetilde{\mu}$ satisfies this inequality. Before proceeding further, it is convenient to work with a quasi-value function of the firm defined in terms of μ as opposed to J_t. Define the quasi-value function $\mathcal{Q}(\mu)$ as:

$$\mathcal{Q}(\mu) = \frac{a}{1-\beta(1-\delta)\left[1-\eta(1-\mu)\right]}.$$

By construction, $\mathcal{Q}(\mu)$ is the value of the firm as long as the job-finding rate is $1-\mu$ forever. Note that $\mathcal{Q}'(\mu) > 0$ and $\mathcal{Q}''(\mu) > 0$.

Under this quasi-value function and given free entry, $\widetilde{\mu}$ satisfies

$$\mathcal{Q}(\widetilde{\mu}) = \frac{a}{1-\beta(1-\delta)\left(1-\eta+\eta\widetilde{\mu}\right)} = \kappa + \chi\widetilde{\mu}.$$

Since the left hand side is convex and the right hand side linear, since $\widetilde{\mu}$ is the smaller of two solutions to this equation, then

$$\mathcal{Q}'(\widetilde{\mu}) = \frac{a\beta(1-\delta)\eta}{\left[1-\beta(1-\delta)\left(1-\eta+\eta\widetilde{\mu}\right)\right]^2} < \chi.$$

In other words, the LHS cuts the RHS from above. Next, dividing the first equality by the second inequality, we have

$$\widetilde{\mu} < \frac{1}{2}\left[\frac{1-\beta(1-\delta)}{\beta(1-\delta)\eta} + 1 - \frac{\kappa}{\chi}\right]. \tag{D.2}$$

Define:

$$\Xi = \frac{1}{2}\left\{\frac{(1-(1-\delta)\underline{\mu})(1-\eta)}{\eta(1-\delta)\underline{\mu}} - \frac{1-\beta(1-\delta)}{\eta\beta(1-\delta)} + \frac{1}{(1-\delta)} - 1\right\}$$

Assuming that $\beta > \frac{\widetilde{\mu}}{\eta\widetilde{\mu}+1-\eta}$, it can be shown that $\Xi > 0$.[25] Thus, as required:

$$\widetilde{\mu} < \frac{1}{2}\left[\frac{1-\beta(1-\delta)}{\beta(1-\delta)\eta} + 1 - \frac{\kappa}{\chi}\right] + \Xi = \frac{1}{2}\left[\frac{\left(1-(1-\delta)\underline{\mu}\right)(1-\eta)}{\eta(1-\delta)\underline{\mu}} + \frac{1}{(1-\delta)} - \frac{\kappa}{\chi}\right].$$

□

It was already clear that given a $\mu_{t+1} \in I^1$, there exists a unique μ_t which could have led there. In addition, this Lemma shows that given any μ_t, there exists at most one $\mu_{t+1} \in I^1$ is consistent with equilibrium.

Corollary 2. *Let $I^2 = F^1(I^1)$ and let $M^2(\mu)$ be the inverse of this function. Then $M^2(\mu^1) = M^1(\mu^1) = \underline{\mu}$.*

Since F^1 is increasing and continuous, its inverse M^2 exists and is increasing and continuous. Consequently, $F^1(I^1)$ maps into an interval $(\mu^1, \mu^2]$. Further since $\mu^1 = F^1(\underline{\mu})$, then $M^2(\mu^1) = \underline{\mu}$.

Lemma 3. $\mu^2 = F^1(\mu^1) < \widetilde{\mu}$

Proof. Since $\Theta^1(\underline{\mu}) = 1 - \underline{\mu}$ and Θ^1 is increasing, we have $\Theta^1(\mu^1) > 1 - \underline{\mu} > 1 - \widetilde{\mu}$. It follows that:

$$\frac{1}{\chi}\left[a-\kappa+\beta(1-\delta)(1-\eta\Theta^1(\mu^1))(\kappa+\chi\widetilde{\mu})\right] \quad < \quad \frac{1}{\chi}\left[a-\kappa+\beta(1-\delta)(1-\eta(1-\widetilde{\mu}))(\kappa+\chi\widetilde{\mu})\right].$$

[25] Note that this assumption is a condition on an endogenous variable, $\widetilde{\mu}$ and can be rewritten as $\widetilde{\mu} < \frac{1-\eta}{\beta^{-1}-\eta}$. Nonetheless, it is a weak condition: for any $\widetilde{\mu} < 1$, it is satisfied for β sufficiently close to 1.

Then, from Corollary 1, since $\mu^1 < \widetilde{\mu}$:

$$\begin{aligned}
F^1(\mu^1) &= \frac{1}{\chi}\left[a - \kappa + \beta(1-\delta)(1-\eta\Theta^1(\mu^1))(\kappa + \chi\mu^1)\right] \\
&< \frac{1}{\chi}\left[a - \kappa + \beta(1-\delta)(1-\eta(1-\widetilde{\mu}))(\kappa + \chi\widetilde{\mu})\right] \\
&= \widetilde{\mu}.
\end{aligned}$$

□

Lemma 4. *Define* $\Theta^2(\mu) : I^2 \to [0,1]$ *as:*

$$\Theta^2(\mu) := 1 - M^2(\mu)\frac{1-(1-\delta)\mu}{1-(1-\delta)M^2(\mu)}.$$

Then,

$$\frac{\partial \Theta^2(\mu)}{\partial \mu} \leq \frac{(1-\delta)M^2(\mu)}{1-(1-\delta)M^2(\mu)}.$$

Proof.

$$\begin{aligned}
\frac{\partial \Theta^2(\mu)}{\partial \mu} &= M^2(\mu)\frac{(1-\delta)}{1-(1-\delta)M^2(\mu)} - \frac{\partial M^2(\mu)}{\partial \mu}\left[1 + \frac{(1-\delta)\left(1-(1-\delta)\mu\right)M^2(\mu)}{\left[1-(1-\delta)M^2(\mu)\right]^2}\right] \\
&\leq M^2(\mu)\frac{(1-\delta)}{1-(1-\delta)M^2(\mu)}.
\end{aligned}$$

where the inequality comes because $M^2(\mu)$ is increasing and the expression in square brackets is positive. □

We are now ready to characterise equilibrium in the entire convalescent region.

Lemma 5 (Induction Step)**.** *Suppose the functions* $\Theta^n(\mu)$*,* $M^n(\mu)$ *are defined on some interval* $I^n = [\mu^{n-1}, \mu^n]$ *and* $M^{n-1}(\mu_{T-n+1})$ *is defined on an interval* $I^{n-1} = [\mu^{n-2}, \mu^{n-1}]$*, with* $\underline{\mu} < \mu^{n-2} < \mu^n < \widetilde{\mu}$*, and that these functions satisfy*

$$\begin{aligned}
\Theta^n(\mu) &= 1 - M^n(\mu)\frac{1-(1-\delta)\mu}{1-(1-\delta)M(\mu)}, \\
\frac{\partial \Theta^n(\mu)}{\partial \mu} &< \frac{(1-\delta)M^n(\mu)}{1-(1-\delta)M^n(\mu)}, \\
M^n(I^n) &= I^{n-1}, \\
M^n(\mu^{n-1}) &= M^{n-1}(\mu^{n-1}) = \mu^{n-2}.
\end{aligned}$$

Then, for β *sufficiently close to 1, we have the following results:*

1. *The function*

$$F^n(\mu) := \frac{1}{\chi}\left[a - \kappa + \beta(1-\delta)(1-\eta\Theta^n(\mu))(\kappa + \chi\mu)\right]$$

   is monotonically increasing in μ *for* $\mu \leq \widetilde{\mu}$*.*

2. *Let $I^{n+1} = F^n(I^n)$ and let $M^{n+1}(\mu)$ be the inverse of this function. Then $M^{n+1}(\mu^n) = M^n(\mu^n) = \mu^{n-1}$.*

3. *$I^{n+1} = [\mu^n, \mu^{n+1}]$ with $\mu^{n+1} < \widetilde{\mu}$.*

4. *Define $\Theta^{n+1}(\mu)$ on I^{n+1} by*

$$\Theta^{n+1}(\mu) = 1 - M^{n+1}(\mu)\frac{1-(1-\delta)\mu}{1-(1-\delta)M^{n+1}(\mu)}.$$

The derivative of this function satisfies

$$\frac{\partial\Theta^{n+1}(\mu)}{\partial\mu} < \frac{(1-\delta)M^{n+1}(\mu)}{1-(1-\delta)M^{n+1}(\mu)}.$$

Proof. (1.) The derivative of $F^n(\mu)$ is

$$\begin{aligned}\frac{\partial F^n(\mu)}{\partial\mu} &= \frac{\beta(1-\delta)}{\chi}\left[-\eta\frac{\partial\Theta^n(\mu)}{\partial\mu}(\kappa+\chi\mu)+\chi(1-\eta\Theta^n(\mu))\right]\\ &> \frac{\beta(1-\delta)}{\chi}\left[-\eta\frac{(1-\delta)M^n(\mu)}{1-(1-\delta)M^n(\mu)}(\kappa+\chi\mu)+\chi(1-\eta\Theta^n(\mu))\right].\end{aligned}$$

Substituting in the definition of Θ^n and rearranging, we see that this expression will be positive provided that

$$\mu < \frac{1}{2}\left[\frac{1-\eta}{\eta}\frac{(1-(1-\delta)M^n(\mu))}{(1-\delta)M^n(\mu)}+\frac{1}{(1-\delta)}-\frac{\kappa}{\chi}\right].$$

By the same logic as in Lemma 2, for β sufficiently close to 1, this is satisfied for any $\mu \leq \widetilde{\mu}$, since we have $M^n(\mu) \leq \widetilde{\mu}$. So $F^n(\mu)$ is increasing, and hence invertible, for $\mu < \widetilde{\mu}$. Let $M^{n+1}(\mu)$ be the inverse of this function.

(2.) We have

$$\begin{aligned}M^n(\mu^{n-1}) &= M^{n-1}(\mu^{n-1}),\\ \Theta^n(\mu^{n-1}) &= \Theta^{n-1}(\mu^{n-1}),\\ F^n(\mu^{n-1}) &= F^{n-1}(\mu^{n-1}) = \mu^n \text{ by definition of } \mu^n,\\ M^{n+1}(\mu^n) &= M^n(\mu^n).\end{aligned}$$

(3.) Since F^n is a continuous, increasing function, the image of the interval $[\mu^{n-1}, \mu^n]$ under F^n must be an interval $[\mu^n, \mu^{n+1}]$. (We have already shown that $F^n(\mu^{n-1}) = \mu^n$.) We need to show that

$\mu^{n+1} = F^n(\mu^n) < \widetilde{\mu}$. We know that $\widetilde{\mu} \geq M^n(\mu^n)$. Then, it must be true that

$$\begin{aligned}
1 - \widetilde{\mu} &< 1 - M^n(\mu^n) \\
&= 1 - M^n(\mu^n)\frac{1-(1-\delta)\mu^n}{1-(1-\delta)\mu^n} \\
&< 1 - M^n(\mu^n)\frac{1-(1-\delta)\mu^n}{1-(1-\delta)M^n(\mu^n)} \\
&= \Theta^n(\mu^n).
\end{aligned}$$

Then, by the same logic as in Lemma 3 we have $F^n(\mu^n) < \widetilde{\mu}$. So we have shown that $I^{n+1} \subset [\underline{\mu}, \widetilde{\mu}]$.

(4.) The bound on the derivative is established in the same way as Lemma 4.

□

Lemma 6. $\lim_{n\to\infty} \mu^n \to \widetilde{\mu}$.

Proof. We have shown that $\{\mu^n\}$ is an increasing sequence bounded above by $\widetilde{\mu}$; thus by the Monotone Convergence Theorem, its limit μ^∞ exists, and $\mu^\infty \leq \widetilde{\mu}$. Suppose by contradiction that $\mu^\infty < \widetilde{\mu}$. Then μ^∞ must be a steady state. But by definition, $\widetilde{\mu}$ is the smallest slack steady state. So we must have $\mu^\infty = \widetilde{\mu}$. □

Finally, we prove that recoveries can be arbitrarily slow, i.e. for any $T \in \mathbb{N}$, there exists $\varepsilon > 0$ such that if $\mu_0 \in (\widetilde{\mu} - \varepsilon, \widetilde{\mu})$, $\mu_t > 0$ for all $t < T$. Fix $\delta > 0, T \in \mathbb{N}$ and let n be the smallest integer such that $\mu^n \geq \widetilde{\mu} - \delta$ (this exists, since $\mu^n \to \widetilde{\mu}$ and $\delta > 0$. Set $\varepsilon = \widetilde{\mu} - \mu^{n+T}$. Take any $\mu_0 \in (\widetilde{\mu} - \varepsilon, \widetilde{\mu}) = (\mu^{n+T}, \widetilde{\mu})$. Then $\mu_0 \in (\mu^{m-1}, \mu^m]$ for some $m > n + T + 1$. We know from 2 that $\mu_T \in (\mu^{m-T-1}, \mu^{m-T}]$. In particular,

$$\mu_T > \mu^{m-T-1} > \mu^n \geq \widetilde{\mu} - \delta > \mu_0 - \delta.$$

Finally, since $\{\mu_t\}$ is monotonically decreasing, we have $\mu_t > \mu_0 - \delta$ for all $t < T$, as claimed. Next, note that the first part of the lemma is a special case of the second part with $\delta = \widetilde{\mu}$.

E Proof of Proposition 3

To see this more formally, note that any trajectory which starts to the right of $\underline{\mu}$ and reached full employment at some date T has to be at $\underline{\mu}$ at date $T-2$. But Proposition 2 showed that *all* trajectories that reach $\underline{\mu}$ lie entirely within the convalescent region. It follows that if the economy starts in the stagnant region - defined as the set $[\widetilde{\mu}, 1]$ - it can never converge to full employment - this region is an *unemployment trap*.

F Planning Problems

F.1 Discretion

We consider a planner who solves the problem

$$\mathbb{L}\left(\mu_t, W_{t-1} \mid \beta_t\right) = \min_{W_t, u_t, P_t, \theta, \mu_{t+1}} u_t^2 + \lambda \left(\frac{W_t}{W_{t-1}} - 1 \right)^2 + \beta_t \mathbb{L}\left(\mu_{t+1}, W_t \mid \beta_{t+1}\right),$$

s.t.

$$\begin{aligned}
W_t &= \max\left\{\varphi W_{t-1}, P_t \omega^*\left(\mu_t\right)\right\}, \\
\mu_{t+1} &= \frac{u_t}{1-(1-\delta)\left(1-u_t\right)}, \\
\frac{\kappa}{f_t} + \chi\mu_t &= A - \frac{W_t}{P_t} + \beta_t (1-\delta)\left(\frac{\kappa}{f_{t+1}} + \chi\mu_{t+1}\right), \\
\mu_{t+1} &= \frac{1-q_t}{1+(1-\delta)\left(1-q_t-\mu_t\right)}, \\
f_t &= \min\left\{\frac{1}{\theta_t}, 1\right\}, \qquad q_t = \min\left\{1, \theta_t\right\}, \qquad P_t \leq \frac{1}{\beta_t} P_{t+1}.
\end{aligned}$$

where $\beta_0 > 1$, $\beta_t = \beta \in (0,1)$ for all $t > 0$. We will show that the solution to this problem is

$$P_t \leq \frac{W_{t-1}}{\omega^*\left(\mu_t\right)}, \; i_t \geq 0 \qquad \text{with at least one equality },$$

and if $P_t = W_{t-1}/\omega^*\left(\mu_t\right)$, then $\mu_{t+1} = \mathcal{M}\left(\mu_t\right)$.

Suppose that monetary policy indeed follows the policy described above from date $t+1$ onwards. This implies that the loss $\mathbb{L}\left(\mu_{t+1}, \cdot \mid \beta_{t+1}\right)$ is increasing in μ_{t+1}, since a higher value of μ_{t+1} implies a higher value of μ_{t+k} for all $k > 1$, and thus a larger per-period loss u^2. Thus at date t, choosing $\mu_{t+1} > \mathcal{M}\left(\mu_t\right)$ delivers a strictly higher loss than choosing $\mu_{t+1} = \mathcal{M}\left(\mu_t\right)$ and $W_t = W_{t-1}$. If setting $P_t = W_{t-1}/\omega^*\left(\mu_t\right)$ and $\mu_{t+1} = \mathcal{M}\left(\mu_t\right)$ does not violate the ZLB, it is therefore optimal. Setting a price $P_t < W_{t-1}/\omega^*\left(\mu_t\right)$ is not optimal as it results in a higher real wage, fewer vacancies and thus a higher μ_{t+1} which entails a higher loss, as we have just argued. Finally, because real wages cannot be lower than their natural level, so setting a higher price $P_t > W_{t-1}/\omega^*\left(\mu_t\right)$ not only still leads to $\mu_{t+1} = \mathcal{M}\left(\mu_t\right)$ but also gives rise to a non-zero wage inflation $W_t > W_{t-1}$ as nominal wages increase one-for-one with prices in this region. It follows that any positive level of wage inflation leads to a strictly higher loss. Thus, implementing zero nominal wage inflation and replicating the natural allocation is optimal under discretion as long as the ZLB does not bind.

However, if setting $P_t = W_{t-1}/\omega^*\left(\mu_t\right)$ and $\mu_{t+1} = \mathcal{M}\left(\mu_t\right)$ does violate the ZLB at date t, then optimal policy under discretion will implement $i_t = 0$ and $P_t < W_{t-1}/\omega^*\left(\mu_t\right)$. It can never be optimal to set $i_t = 0$ and $P_t > W_{t-1}/\omega^*\left(\mu_t\right)$: whenever it is feasible to do so, implementing the natural real wage and zero wage inflation is also feasible and optimal. Setting $i_t = 0$ and $P_t > W_{t-1}/\omega^*\left(\mu_t\right)$ would result in positive wage inflation $W_t > W_{t-1}$ and real wages at their flexible price level $W_t/P_t = \omega^*\left(\mu_t\right)$. This is dominated by setting $P_t = W_{t-1}/\omega^*\left(\mu_t\right)$, which results in the same level of real wage and thus the same μ_{t+1}, but zero nominal wage inflation. Implementing this lower price level does not violate the ZLB as it entails setting a higher interest rate $i_t > 0$. Thus, whenever it is constrained optimal to set $i_t = 0$, it must be that $P_t < W_{t-1}/\omega^*\left(\mu_t\right)$. $\square$

F.2 Commitment

We now show that UMP is optimal under commitment for a planner who seeks to minimise

$$\sum_{t=0}^{\infty}\prod_{j=0}^{t}\beta_{t-j}\left\{u_t^2+\lambda\left(\frac{W_t}{W_{t-1}}-1\right)^2\right\},$$

for $\lambda > 0$ sufficiently small. Under UMP, $W_1/W_0 = \beta_0\omega^*_{\text{fe}}/\omega_0^{ump} > 1$ at date 1; wage inflation equals zero at all other dates, and unemployment remains at zero at all dates. Thus, UMP attains the loss $\beta_0\lambda\left(\beta_0\omega^*_{\text{fe}}/\omega_0^{ump}-1\right)^2$. Since unemployment remains at zero under UMP, to show that this loss is smaller than the loss associated with any other policy, it suffices to compare it to other policies which involve less nominal wage inflation at date 1, i.e.,

$$\beta_0\frac{\omega^*_{\text{fe}}}{\omega_0^{ump}} > \frac{W_1}{W_0} = \frac{P_1}{P_0}\frac{\omega_1}{\omega_0} \geq \beta_0\frac{\omega_1}{\omega_0}.$$

That is, under one of these alternative policies, either $\omega_0 > \omega_0^{ump}$, or $\omega_1 < \omega^*_{\text{fe}}$, or both. Either one of these conditions implies that we must have $\mu_1 \geq \underline{\mu}$. First suppose $\omega_1 < \omega^*_{\text{fe}}$. Since $\omega_1 \geq \omega^*(\mu_1)$ and $\omega^*(\mu) = \omega^*_{\text{fe}}$ for all $\mu < \underline{\mu}$, this directly implies $\mu_1 \geq \underline{\mu}$. Next, suppose $\omega_0 > \omega_0^{ump}$. The free entry condition at date 0 is

$$J_0 = A - \omega_0 + \beta_0(1-\delta)J_1 \quad \geq \quad \kappa = A - \omega_0^{ump} + \beta_0(1-\delta)J_{min},$$

which can be rearranged to get:

$$J_1 - J_{min} \geq \frac{\omega_0 - \omega_0^{ump}}{\beta_0(1-\delta)} > 0.$$

So we have $J_1 > J_{min}$. Given the definitions of these variables,

$$J_1 = \sum_{t=1}^{\infty}\beta^{t-1}(1-\delta)^{t-1}(A-\omega_t), \quad \text{and} \quad J_{min} = \frac{A-\omega^*_{\text{fe}}}{1-\beta(1-\delta)},$$

this implies that there must be some date $T \geq 1$ at which $\mu_T \geq \underline{\mu}$ for the first time. (If this were not true, and $\mu_t < \underline{\mu}$ for all t, then since $\omega_t \geq \omega^*(\mu_t)$ and $\omega^*(\mu) = \omega^*_{\text{fe}}$ for all $\mu < \underline{\mu}$, we would have $J_1 = J_{min}$.) We will now show that we must have $T = 1$. Suppose by contradiction that $T > 1$; then $\mu_T > 0$ implies $\theta_{T-1} < 1$ and so the date $T-1$ free entry condition is $J_{T-1} \leq \kappa < J_{min}$. But then, since $\mu_t < \underline{\mu}$ and $\omega_t \geq \omega^*_{\text{fe}}$ for all $1 \leq t < T$, we have

$$J_1 = \sum_{t=1}^{T-2}[\beta(1-\delta)]^{t-1}(A-\omega_t) + [\beta(1-\delta)]^{T-1}J_{T-1} < J_{min},$$

which contradicts the condition that $J_1 > J_{min}$. So we must have $\mu_1 \geq \underline{\mu}$ under any policy involving less date 1 wage inflation than UMP, as claimed above. Thus, date 0 unemployment must be at least $u(\underline{\mu}) \equiv \delta\underline{\mu}/[1-\underline{\mu}\,(1-\delta)]$, and the loss from this alternative poicy must be at least $u(\underline{\mu})^2$. Thus, UMP will be preferred to this alternative policy as long as $\beta_0 \lambda \left(\beta_0 \omega^*_{\text{fe}}/\omega_0^{ump} - 1\right)^2 \leq u(\underline{\mu})^2$ which is the case for λ small enough:

$$0 \leq \lambda \leq \frac{u\left(\underline{\mu}\right)^2}{\left(\beta_0 \frac{\omega^*_{\text{fe}}}{\omega^*_0} - 1\right)^2}.$$

G Properties of $J_0(\beta_0)$

Suppose that the economy remains at full employment steady state even after the shock $\beta_0 > 1$. There are two cases to consider. First, suppose that the ZLB does not bind at date 0. Then monetary policy is unconstrained in all periods, and nominal wages and prices remain constant. From (14), we have $1 + i_t = P_1/(P_0\beta_0) = 1/\beta_0$. When $\beta_0 > 1$, this would imply a negative nominal interest rate, violating the ZLB. Thus, when $\beta_0 > 1$, monetary policy is constrained at date 0 and we have $P_0 = P_1/\beta_0$. Since the economy returns to full employment after date 0, real wages will equal ω^*_{fe} at all dates $t \geq 1$. Iterating forward $P_t = \min\left\{W_{t-1}/\omega^*(\mu_t), P_{t+1}\beta_t\right\}$, it follows that prices and nominal wages remain constant thereafter and the ZLB does not bind after date 0. In particular, since $W_1 = W_0$, we have:

$$\omega_0 = \frac{W_0}{W_1}\frac{P_1}{P_0}\omega_1 = \beta_0 \omega^*_{\text{fe}}.$$

Using this in the expression for J_0 we have:

$$J_0 = A - \beta_0 \omega^*_{\text{fe}} + \beta_0(1-\delta)J_{min}.$$

The full employment steady state Nash wage equals

$$\omega^*_{\text{fe}} = \frac{\eta}{1-\beta(1-\delta)(1-\eta)}A + \frac{[1-\beta(1-\delta)](1-\eta)}{1-\beta(1-\delta)(1-\eta)}b.$$

So

$$\frac{\partial J}{\partial \beta_0} = -\omega^*_{\text{fe}} + (1-\delta)J_{min} = -\frac{\eta}{1-\beta(1-\delta)(1-\eta)}A - \frac{[1-\beta(1-\delta)](1-\eta)}{1-\beta(1-\delta)(1-\eta)}b + (1-\delta)\frac{(1-\eta)(A-b)}{1-\beta(1-\delta)(1-\eta)},$$

which is negative provided that $A\left[1-\delta-\eta/(1-\eta)\right] - [2-\delta-\beta(1-\delta)]b < 0$. By Assumption 2, both terms are negative, so this condition is satisfied.

H Proof of Proposition 4

First we show that a one-period hiring freeze takes the economy either to the convalescent or to the stagnant region.

Lemma 7. *Starting from full employment, a one period hiring freeze takes the economy out of the healthy region:* $\mu_R = 1/(2-\delta) > \underline{\mu}$.

Proof. We prove the Lemma by proving the contrapositive. The first thing to note is that $\mu_R :=$

$1/(2-\delta) > 0.5$ since $0 < 1-\delta < 1$. Recall that $\underline{\mu} = (J_{min} - \kappa)/\chi$. Suppose $\underline{\mu} \geq \mu_R$. This implies that $\underline{\mu}$ must also be greater than 0.5. In this case, no interior steady state can exist. Recall that any interior steady state solves:

$$\begin{aligned}
\kappa + \chi\mu &= \mathcal{Q}(\mu) \\
&= \frac{a}{1-\beta(1-\delta)[1-\eta(1-\mu)]} \\
&= \frac{a}{1-\beta(1-\delta)(1-\eta)} \frac{1-\beta(1-\delta)(1-\eta)}{1-\beta(1-\delta)[1-\eta(1-\mu)]} \\
&= J_{min}\frac{1}{1-e\mu},
\end{aligned}$$

where, as before $e = \beta(1-\delta)\eta/[1-\beta(1-\delta)(1-\eta)]$.

Thus interior steady states solve:

$$\Omega(\mu) := \frac{J_{min}}{1-e\mu} - \kappa - \chi\mu = 0.$$

We show that this is not possible if $\underline{\mu} > \mu_R$. In particular, we have $\Omega(\mu) > 0$ for all $\mu \in [0,1]$. First, we show that $e > 1/2$ and $\chi < 2(J_{min} - \kappa)$. Notice that e can also be rewritten as:

$$e = \frac{1}{1 + \frac{1-\beta(1-\delta)}{\beta(1-\delta)\eta}} > \frac{1}{1+\frac{\eta}{\eta}} = \frac{1}{2},$$

where the inequality follows since $\eta > [1-\beta(1-\delta))]/\beta(1-\delta)$ by Assumption 2. Thus, $e > \frac{1}{2}$. To see that $\chi < 2(J_{min} - \kappa)$, note that from the definition of $\underline{\mu}$:

$$\chi = \frac{J_{min} - \kappa}{\underline{\mu}} < 2(J_{min} - \kappa),$$

since $\underline{\mu} > 0.5$ by assumption.

Fix $\kappa \in [0, J_{min})$, $\mu \in [0,1]$. Even though we have shown above that $e > 1/2$ and $\chi < 2(J_{min} - \kappa)$, for a moment, set $e = 1/2$, $\chi = 2(J_{min} - \kappa)$. We claim that

$$\mathcal{Q}(\mu) = \frac{J_{min}}{1-e\mu} \geq \kappa + \chi\mu = \kappa + 2(J_{min} - \kappa)\mu,$$

with strict inequality unless $\kappa = 0$ and $\mu = 1$, in which case the expression holds with equality. When $\kappa = 0$, the RHS becomes $2J_{min}\mu$, and the LHS and RHS are only equal for $\mu = 1$. For any $\mu < 1$, the LHS is larger. When $\kappa > 0$, the RHS is strictly lower for any $\mu > 1/2$. Thus for any $\kappa \in [0, J_{min}]$, the inequality holds for all $\mu \in [0,1)$. Finally, for any $\mu \leq 1/2$, the inequality clearly holds since the LHS is greater than J_{min}, and the RHS smaller than J_{min}.

Next, suppose $e > 1/2$ and $\chi < 2(J_{min} - \kappa)$. If $\mu = 0$, this does not change the inequality, which still holds strictly (since $\mu \neq 1$). If $\mu > 0$, this strictly increases the LHS and strictly decreases the RHS. Thus the expression is still satisfied with strict inequality. Thus we have $\Omega(\mu) > 0$ for all $\mu \in [0,1]$, and there is no interior steady state. Since we have shown that $\underline{\mu} \geq \mu_R$ implies there exists no interior steady state, it follows that if there exist multiple interior steady states, we must have $\underline{\mu} < \mu_R$. $\square$

Next we need to prove two lemmas. The first states that wages are lower in the convalescent region than at full employment. We need this result to show that prices will be higher in the convalescent region.

Lemma 8. *$\omega^*(\mu_t) < \omega^*_{\text{fe}}$ if $\mu_t \in (\underline{\mu}, \tilde{\mu})$.*

Proof. We know that $M(\mu_t) < \mu_t$ if $\mu_t \in (\underline{\mu}, \tilde{\mu})$.

$$\begin{aligned}
\omega^*(\mu_t) &= A - (\kappa + \chi\mu_t) + \beta(1-\delta)[\kappa + \chi M(\mu_t)] \\
&= A - \beta(1-\delta)\chi(\mu_t - M(\mu_t)) - (1-\beta(1-\delta))(\kappa + \chi\mu_t) \\
&< A - (1-\beta(1-\delta))(\kappa + \chi\mu_t) \\
&< A - (1-\beta(1-\delta))(\kappa + \chi\underline{\mu}) = \omega^*_{\text{fe}}.
\end{aligned}$$

$\square$

Lemma 9. *Under Assumption 2, $\frac{W_t}{P_t} > (1-\delta)[\kappa + \chi\mu_t]$.*

Proof. We know that $\frac{W_t}{P_t} \geq \omega^*(\mu_t)$ by definition, so it suffices to show that $\omega^*(\mu_t) > (1-\delta)[\kappa + \chi\mu_t]$. In the flexible wage benchmark we have

$$\omega_t = \eta A + (1-\eta)b + \beta(1-\delta)q_{t+1}J_{t+1} \geq \eta A + (1-\eta)b > (1-\delta)(\kappa + \chi\mu_t)$$

for any $\mu_t \in [0,1]$, given Assumption 2. $\square$

Finally, we need to characterise dynamics of the economy starting at date 1, once the shock has abated. Under neutral monetary policy, if the ZLB never binds, allocations are (by definition) equal to those in the flexible wage benchmark. The following is immediate.

Lemma 10. *If $\mu_1 \geq \tilde{\mu}$, the economy never returns to the full employment steady state.*

Proof. If the ZLB never binds, allocations are equivalent to those in the flexible wage benchmark, and we know that the economy never returns to steady state. It only remains to show that the ZLB can never help the economy converge to the full employment steady state. Suppose by contradiction that the economy converges to the full employment steady state. Let μ_t^R, μ_t^N denote allocations in the flexible wage benchmark and in the nominal economy, respectively, given the initial condition $\mu_1 \geq \tilde{\mu}$. Let $T \geq 1$ be the first date at which $\mu_t^N < \mu_t^R$ (there must be some such date, since in the long run $\mu_t^N = 0, \mu_t^R > 0$, by assumption). Then we have

$$\begin{aligned}
J_{T-1}^N = \kappa + \chi\mu_{T-1}^N = \kappa + \chi\mu_{T-1}^R = J_{T-1}^R, \\
J_T^N = \kappa + \chi\mu_T^N < \kappa + \chi\mu_T^R = J_T^R.
\end{aligned}$$

This implies that real wages are higher at date $T-1$ in the flexible wage benchmark than in the nominal economy:

$$\begin{aligned} J_{T-1}^N &= J_{T-1}^R, \\ A - \omega_T^N + \beta(1-\delta)J_T^N &= A - \omega_T^R + \beta(1-\delta)J_T^R, \\ \omega_t^R - \omega_t^N &= \beta(1-\delta)(J_T^R - J_T^N) > 0. \end{aligned}$$

This is a contradiction - given the downward nominal wage rigidities, wages are always weakly higher than in the flexible wage benchmark. Thus the economy cannot converge to the full employment steady state. $\square$

We are now ready to prove Proposition 4. Part 1. follows for the same reasons as in the previous lemmas. Define the function

$$B(\mu) = \frac{A-\kappa}{\omega(\mu) - (1-\delta)(\kappa + \chi\mu)}$$

on $(\underline{\mu}, \mu_R]$, where $\omega(\mu_1)$ denotes the prevailing real wage at date 1 as a function of μ_1. It is straightforward to show that $\omega(\mu_1)$ is continuous, and thus B is continuous. We have $B(\underline{\mu}) = \underline{\beta}$. Define $\overline{\beta} := B(\mu_R)$. If $\beta_0 > \overline{\beta}$, then if $\mu_1 = \mu_R$, we have

$$J_0 = A - \beta_0\omega(\mu_R) + \beta_0(1-\delta)(\kappa + \chi\mu_R) < \kappa,$$

thus $\theta_0 = 0$, which is consistent with $\mu_1 = \mu_R$. If instead $\beta_0 \in (\underline{\beta}, \overline{\beta})$, then there exists $\mu \in (\underline{\mu}, \mu_R)$ such that $B(\mu) = \beta_0$, and a corresponding $\theta_0 = 1 - \mu_1/[1-(1-\delta)\mu_1]$. Then we have

$$J_0 = \kappa = A - \beta_0\omega(\mu_1) + \beta_0(1-\delta)(\kappa + \chi\mu_1),$$

and firms are indifferent between posting any number of vacancies; thus $\theta_0 \in [0,1]$ can indeed be an equilibrium. Finally, the fact that the economy does not return to full employment if it is thrown into the stagnant region follows from Lemma 10.

I Proof of Proposition 5

Under UMP, at date 0, the value of a filled vacancy J_0 is given by:

$$J_0 = A - \omega_0^{\text{ump}}(\beta_0) + \beta_0(1-\delta)J_{min} = \kappa,$$

where we have used the definition of $\omega_0^{\text{ump}}(\beta_0)$. Since, $\mu_0 = 0$, the free entry condition is satisfied with $\theta_0 = 1$. This implies that $q_0 = 1$ and the economy remains at full-employment.Next, note that the date 0 Fisher equation implies:

$$1 + i_0 = \beta_0^{-1}\frac{P_1}{P_0} = 1,$$

while at subsequent dates, we have:

$$1 + i_t = \beta^{-1}\frac{P_{t+1}}{P_t} = \beta^{-1} > 1.$$

Thus, the ZLB constraint is satisfied at all dates (and binds only at date 0). The DNWR constraint is satisfied at all dates since $W_0 = W_{-1}$ and $W_0 > P_0\omega^*(\mu_0)$, while for $t \geq 1$, $W_t \geq W_{t-1}$ and $W_t = P_t\omega^*(\mu_t)$.