Discussion of *The Intertemporal Keynesian Cross*

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January 5, 2018

What do the authors do

- decompose the effect of shocks on aggregate variables in a class of models with nominal rigidities into *PE* and *GE* effects

$$dY = \underbrace{\mathbf{M}}_{\text{MPC matrix}} dY + \underbrace{\partial Y}_{\text{PE}}$$

- argue that **M** is good for:
  1. characterizing/understanding/checking determinacy in potentially complicated models
  2. model validation: comparing model implied **M** with **M** in data
  3. understand when heterogeneity matters for aggregate outcomes
  4. importantly, build intuition about functioning of heterogeneous agent NK models

Discussion

1. ARS find:
   - can get determinacy even with a peg
   - more likely to get determinacy when income risk is procyclical
   - i use stylized model to understand these findings

2. How to compute **M** and use it for
   - model validation (i.e. how to compare to data)
   - check determinacy

3. Some math questions

A simple CARA example

- unit mass of agents indexed by $i \in [0, 1]$
- constant net real interest rate: $r > 0$
- each agent maximizes:

$$\mathbb{E}_0 \left\{ -\frac{1}{\alpha} \sum_{t=0}^{\infty} e^{-\rho t - \alpha c_{i,t}} \right\}$$

s.t.

$$c_{i,t} + a_{i,t} = (1 + r)\, a_{i,t-1} + y_{i,t}$$

$$y_{i,t} \sim N\Big(y_t, \sigma^2(y_t)\Big)$$

CARA continued...

- Guess and verify that the *consumption function* is given by:

$$c_{i,t} = x_t + m\left[(1+r)a_{i,t-1} + y_{i,t}\right]$$

where $m = \frac{r}{1+r}$ and

$$x_t = \sum_{s=1}^{\infty} \frac{1}{(1+r)^s} \left\{ \varphi(r) + m y_{t+s} - \frac{\alpha m^2}{2} \sigma^2(y_{t+s+1}) \right\}$$

- Aggregate date 0 consumption:

$$c_0 = x_0 + m y_0$$

CARA continued...determinacy

- Imposing GE: ($c_t = y_t$), can summarize model as:

$$y_t = \varphi(r) + y_{t+1} - \underbrace{\frac{\alpha r^2}{2\left(1+r\right)^2}\sigma^2\left(y_{t+1}\right)}_{\substack{\text{precautionary} \\ \text{savings}}}$$

- Determinacy depends on linearized equation:

$$\hat{y}_t = \left[1 - \frac{\alpha r^2 \gamma}{2\left(1+r\right)^2}\right]\hat{y}_{t+1}$$

where $\gamma = \frac{d\sigma^2(y)}{dy}$ is the cyclicality of income risk.

acyclical risk $\gamma = 0$

$$\hat{y}_t = \left[1 - \frac{\alpha r^2 \gamma}{2\left(1+r\right)^2}\right] \hat{y}_{t+1}$$

- acyclical risk: $\gamma = 0$

$$1 - \frac{\alpha r^2 \gamma}{2\left(1+r\right)^2} = 1$$

  OR

$$\hat{y}_t = \hat{y}_{t+1}$$

  Standard result: Indeterminacy with a peg

procyclical risk

$$\hat{y}_t = \left[1 - \frac{\alpha r^2 \gamma}{2\left(1+r\right)^2}\right] \hat{y}_{t+1}$$

- procyclical risk: $\gamma > 0$

$$1 - \frac{\alpha r^2 \gamma}{2\left(1+r\right)^2} \in (0,1)$$

Non-Standard result: determinacy even with a peg

- "Discounted Euler eqn" (MNS 2017)

countercyclical risk

$$\hat{y}_t = \left[1 - \frac{\alpha r^2 \gamma}{2\left(1+r\right)^2}\right] \hat{y}_{t+1}$$

- countercyclical risk: $\gamma < 0$

$$1 - \frac{\alpha r^2 \gamma}{2\left(1+r\right)^2} > 1$$

  countercyclical income risk makes indeterminacy more likely

- Challe-Ragot (2014), Ravn-Sterk (2017) and others...
- ARS: far-out columns of **M** matrix are tilted towards the past $\Rightarrow$ Consumption today depends strongly on future y.
- countercyclical risk makes FGP worse?

Finding the $\mathbf{M}$ matrix empirically

- Hard to estimate elements of $\mathbf{M}_{t,s} = \frac{\partial c_t}{\partial y_s}$ because:
  - Broda-Parker type evidence typically only provides $\frac{\partial c_{i,t}}{\partial y_{i,t}}$, at best $\frac{\partial c_{i,t}}{\partial y_{i,t-k}}$ for $k > 0$.
  - could ask $\frac{\partial c_{i,t}}{\partial y_{i,t+k}}$
- In general

$$\mathbb{E}_i \left[\frac{\partial c_{i,t}}{\partial y_{i,t+k}} \right] \neq \mathbb{E}_i \frac{\partial c_{i,t}}{\partial y_{t+k}} = \mathbf{M}_{t,t+k}$$

Finding the **M** matrix empirically

- In CARA economy, if you computed the **M** matrix:

$$\mathbf{M}_{0,t} = \mathbb{E}_i \left[\frac{\partial c_{i,0}}{\partial y_t} \right] = m - \frac{\alpha \gamma m^2}{2} \forall t > 0$$

- But a Broda-Parker type exercise would identify:

$$\frac{\partial c_{i,0}}{\partial y_{i,t}} = m \Rightarrow \mathbb{E}_i \left[\frac{\partial c_{i,0}}{\partial y_{i,t}} \right] = m \neq \mathbf{M}_{0,t}$$

- Unfortunately, this approach would miss out on the interesting part related to cyclicality of risk.

Finding the M matrix computationally

- Computing (approx. truncated) **M** matrix is tedious
- Dumb pseudo-code:
  - Solve for steady state
  - for j=1:J:
    1. consider path of agg. income

$$\mathbf{y}(j) = (y, \cdots, \underbrace{y+\varepsilon}_{j^{th}\text{term}}, y, \cdots)$$

    2. Compute implied real interest rates and taxes
    3. Compute individual decision rules (non-stationary)
    4. Simulate to get sequence of aggregate consumption $\mathbf{c}(j)$
    5. Calculate j^{th} column of **M** matrix

$$\mathbf{M}_{t,j} = \frac{c_t(j) - y}{\epsilon}$$

Finding the **M** matrix computationally

- Characterization requires $J \to \infty$. Even dumb-algorithm above requires one to compute $J + 1$ times.
- Computing Blanchard-Kahn conditions easier but
  - requires approximating evolution of wealth distribution
  - but provides little intuition regarding why determinacy hold/doesn't hold

Is $\mathbf{M}$ necessarily left-stochastic?

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} + \lim_{T\to\infty} \frac{a_T}{(1+r)^T} = \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}$$

- In a Arrow-Debreu eq'm, PDV of consumption = PDV of income
  - $r > 0$ and Transversality condition ensures this
- In Bewley economies, we know that if $\beta(1+r) < 1$, then wealth is bounded.
  - Suppose $\beta(1+r) < 1$, then $\lim_{T\to\infty} a_T < \infty$
  - However, if $r < 0$, the bubble term explodes and PDV of consumption $\neq$ PDV of income.
- Related: Since each element of $\mathbf{M}$ is discounted the risk-free rate, what happens if $r < 0$.
- Also, what if there is no risk-free asset?

Overall..

- very important paper!
- very powerful tools...
- ... but I am not totally clear on how or where to use them in place of the standard Blanchard-Kahn machinery
- would be great if the authors include a discussion about settings in which real interest rates are negative, bubble terms cannot be ruled out etc.

THE END