Optimal Policy Rules in HANK
by
McKay and Wolf

Discussion by Sushant Acharya

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question

should household inequality affect the conduct of cyclical stabilization policy?

methodology

McKay and Wolf presents a sequence-space Jacobian based technique to

- derive a "welfare-based" quadratic loss function incorporating the planner's concern for inequality
- derive a solution to the *optimal* policy problem in the form of a targeting rule
- analyze optimal policy which minimizes some ad-hoc loss functions

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main result

concern for inequality only has a moderate effect on optimal interest rate policy

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- allocations which minimize the McKay-Wolf "welfare based" loss function are **not Pareto optimal**

what does the McKay-Wolf loss function deliver?

insights from a simple risk sharing problem

- **2 agents**: $i \in \{1, 2\}$
- **stochastic endowments**: $y_{i,t} = y_t \zeta_{i,t}$ for $i \in \{1, 2\}$
  - **idiosyncratic risk** $\zeta \in \{\zeta_l, \zeta_h\}$ where $\zeta_l = 1 - \Delta, \zeta_h = 1 + \Delta$ for $\Delta \in (0, 1)$
  - **idiosyncratic risk perfectly negatively correlated**

$$P\left[(\zeta_{1,t}, \zeta_{2,t}) = (\zeta_l, \zeta_h)\right] = P\left[(\zeta_{1,t}, \zeta_{2,t}) = (\zeta_h, \zeta_l)\right] = \frac{1}{2}$$

  - **aggregate risk** $\ln y_t \sim N(0, \sigma_y^2)$ $\qquad y_{1,t} + y_{2,t} = y_t$

Pareto problem

- **Pareto problem**:

$$\max \overline{\varphi}_1 \left\{ \sum_{t=0}^{\infty} \sum_{\zeta_1^t, y^t} \beta^t p(\zeta_1^t, y^t) \ln \left(c_1(\zeta_1^t, y^t) \right) \right\} + \overline{\varphi}_2 \left\{ \sum_{t=0}^{\infty} \sum_{\zeta_2^t, y^t} \beta^t p(\zeta_2^t, y^t) \ln \left(c_2(\zeta_2^t, y^t) \right) \right\}$$

s.t. $\quad c_1(\zeta_1^t, y^t) + c_2(\zeta_2^t, y^t) = y_t$

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- **solution**:

$$\boxed{\frac{\overline{\varphi}_1}{c_1(\zeta_1^t, y^t)} = \frac{\overline{\varphi}_2}{c_2(\zeta_2^t, y^t)}} \Rightarrow \boxed{c_1(\zeta_1^t, y^t) = \frac{\overline{\varphi}_1}{\overline{\varphi}_1 + \overline{\varphi}_2} y_t \text{ and } c_2(\zeta_2^t, y^t) = \frac{\overline{\varphi}_2}{\overline{\varphi}_1 + \overline{\varphi}_2} y_t}$$

full insurance: $c_i(\zeta_i^t, y^t)$ does not depend on realization of ζ_i^t

McKay-Wolf problem

- **MW problem**:

$$\max \left\{ \sum_{t=0}^{\infty} \sum_{\zeta_1^t, y^t} \beta^t p(\zeta_1^t, y^t) \varphi_1(\zeta_1^t) \ln \left(c_1(\zeta_1^t, y^t) \right) \right\} + \left\{ \sum_{t=0}^{\infty} \sum_{\zeta_2^t, y^t} \beta^t p(\zeta_2^t, y^t) \varphi_2(\zeta_2^t) \ln \left(c_2(\zeta_2^t, y^t) \right) \right\}$$

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- MW weights on flow utilities at date t $\varphi_1(\zeta_1^t), \varphi_2(\zeta_2^t)$ can depend on history of idiosyncratic shocks *up to date* t
  - same as Pareto weights only if $\varphi_1(\zeta_1^t)$ and $\varphi_2(\zeta_1^t)$ are constant functions
  - else, planner is maximizing distorted individual preferences

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$$\frac{\zeta_{1,t}}{c_1(\zeta_1^t, 1)} = \frac{\zeta_{2,t}}{c_2(\zeta_2^t, 1)} \qquad \Rightarrow \qquad \boxed{c_i(\zeta_i^t, 1) = \zeta_{i,t}}$$

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MW weights reduce planner's incentive to provide insurance

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- can always find feasible Pareto improvement relative to MW's optimal allocation implications for optimal monetary policy

simpler version of MW model

- households $i \in [0,1]$ with preferences $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_{i,t} - n_t - \frac{\psi}{2} \left(\ln \Pi_t \right)^2 \right\}$
- stochastic income $y_{i,t} = \omega_{i,t} y_t$ where $\omega_{i,t} \in \{\omega_{h,t}, \omega_{l,t}\}$
  - $\omega_{l,t} < \omega_{h,t}$, $\Pr(\omega_{j',t} \mid \omega_{j,t-1}) = \frac{1}{2}$ for any (j, j') and $\frac{\omega_{h,t} + \omega_{l,t}}{2} = 1$
  - $\omega_{i,t}$ can vary with GDP y_t: $\omega_{h,t} = \omega_h y_t^{\gamma}$

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- 0 liquidity, borrowing limit: $c_{h,t} = y_{h,t}, c_{l,t} = y_{l,t}$ and $\frac{1}{2} c_{h,t} + \frac{1}{2} c_{l,t} = y_t$

$$y_{h,t}^{-1} = \beta R_t \mathbb{E}_t \left\{ 0.5 y_{h,t+1}^{-1} + 0.5 y_{l,t}^{-1} \right\}$$

monetary policy controls R_t

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$$y_{h,t}^{-1} = \beta R_t \mathbb{E}_t \left\{ 0.5 y_{h,t+1}^{-1} + 0.5 y_{l,t}^{-1} \right\} \qquad \text{monetary policy controls } R_t$$

- Phillips curve

$$\ln \Pi_t = \beta \ln \Pi_{t+1} + \kappa \left(\log y_t - \log z_t \right) + \varepsilon_t$$

do MW weights lead to meaningfully different answer?

- ☐ RANK $(\omega_{h,t} = \omega_{l,t} = 1)$

$$\boxed{\underbrace{(\widehat{y}_t - \widehat{z}_t)}_{\text{output-gap}} + \underbrace{\lambda \widehat{p}_t}_{\text{price-stability}} = 0}$$

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- TANK (any Pareto weights)

$$(\widehat{y}_t - \widehat{z}_t) + \lambda \widehat{p}_t + \underbrace{\delta \times \widehat{y}_t}_{\substack{\text{distributional} \\ \text{concerns}}} = 0$$

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- TANK (MW weights)

$$(\widehat{y}_t - \widehat{z}_t) + \lambda \widehat{p}_t + \delta^\star \times \widehat{y}_t = 0$$

- MW solution
  - puts less weight on output stabilization than any Pareto problem $0 < \delta^\star < \delta$
  - relative magnitude proportional to steady state inequality

$$\frac{\delta}{\delta^\star} \propto \frac{\omega_h}{\omega_l}$$

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- first-order accurate optimal policy from naive LQ requires efficient steady state
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  - naive LQ gives "accurate solution" but not to the "correct" problem
- ideally, extend methodology to solve QQ problem
  - not trivial since constrained efficient steady state may not exist (Bhandari et al.)

final thoughts

- solution to policy problem using MW weights does not satisfy Pareto optimality
  - can trivially always find alternative allocation which makes all agents better off
- using MW weights $\Rightarrow$ optimal monetary policy biased to be closer to RANK
  - assumptions reduce the planner's motives to provide insurance/ reduce inequality